Volume VI, Issue #1, Fall 2005

UNIVERSITY OF MANITOBA

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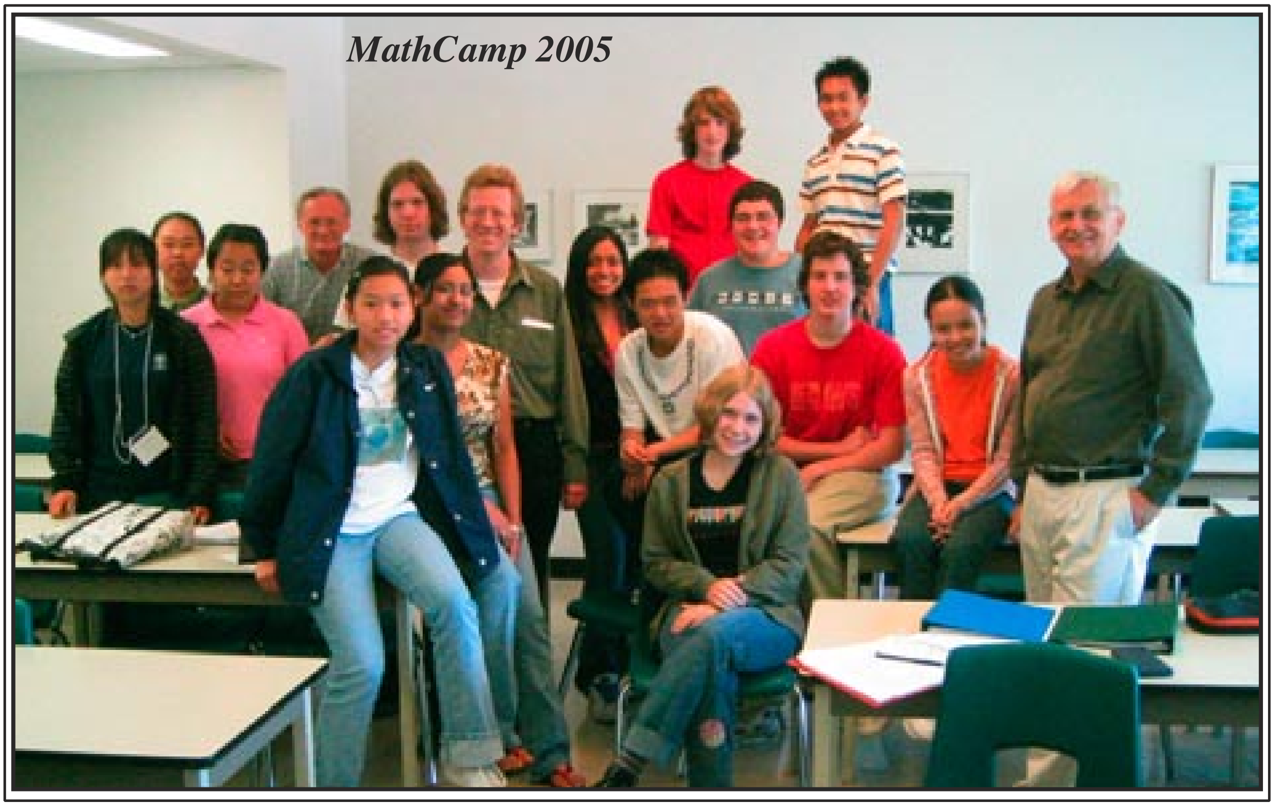
MANITOBA MATH LINKS

BEAUTY · DISCOVERY · PRINCIPLES



(Starting From Left) **Back Row** : Bohyun Seo, Dr. Don Trim, Jason Haydaman, Will Gibson (TA), Bridget Corona, Michael Etkin, Kelly Michaels, Simon Lu
Middle Row: Jumin Jin, Juyoun Seo, Sheena Manghera, Raymond Huynh, Saul Magnusson, Athena Nguyen (TA), Bill Korytowski
Front Row: Lucy-Rose Vuong, Zoë Grabowski

You probably wouldn't be able to convert $5ABD_{16}$ to the decimal system, at least not until you attend a Math Camp. MathCamp 2005 was very unique, different, and was an experience that led us to new friends. Friends, yes! We met new, interesting people from different cultural backgrounds and schools from all over the city. When we first learned of this program, we were expecting...how should we say this...NERDS...but instead we found very cool people who are totally opposite to this.

Other than new friends, this place was far from what we expected. This year we stayed at St. John's College, located at the University of Manitoba, from August 21-25. Aside from staying on campus, we got our own personal rooms and an unlimited amount of food and drink during breakfast, lunch and supper. It was like a buffet and the chefs were very friendly as well. This camp was not all just about mathematical equations and formulas; instead, we had fun playing volleyball, soccer, almost went swimming and to the arcade.

There were many different passionate teachers that taught us new mathematical approaches to problems and were very supportive of our views on new concepts. Aside from the teachers, the TAs, Will and Athena, were very smart and kept us motivated with different puzzles and questions that were mentally challenging. If you ever meet Will Gibson, you must ask him about "hotel infinity". This was mind boggling for all of us. This experience was very interesting and gave us a head start in school.

MathCamp 2005 introduced us to new friends and equations that will, in our opinion, help improve our mathematical approaches to problems. Most people do not think of math as a sport but in reality it is and it is one of the most competitive. If you are good in math, or even if you aren't, this camp is fun and can help express or expand your mathematical horizon. "You don't need to be a nerd to be good at math", a quote from Dr. Gunderson. Thanks to Professors D. Trim and W. Korytowski and also to the TAs, Will and Athena, for a great experience!

Manitoba Math Links is supported by the Manitoba Ministry of Education

MANITOBA MATH LINKS WEB SITE
........www.umanitoba.ca/science/mathematics
(a PDF Version of Math Links is available on this website)

NEED MORE COPIES??????

Feel free to reproduce this newsletter and pass it on.......!

MANITOBA MATH LINKS

The Math Links Newsletter is published by the Mathematics Department Outreach Committee three times a year (Fall, Winter & Spring).

EDITORS:

Prof. D. Gunderson, Editor In Chief
Prof. A. Gumel
Prof. R. Padmanabhan
Prof. R.G. Woods

IMPORTANT DATES TO REMEMBER:

Evening of Excellence - U of M

Tuesday, October 18th

SPEAKERS AVAILABLE

If you are interested in having a faculty member come to your school and speak to students, please contact our Managing Editor at *kangass@cc.umanitoba.ca* or phone 474-8703.

CAREER PROFILES IN MATHEMATICS

The following web sites contain short descriptions of jobs held by individuals with a mathematics degree and a wide variety of backgrounds:
http://www.siam.org/careers/career2.htm#numberfive
http://www.maa.org/careers/index.html

A NOTE FROM THE EDITORS:

We welcome comments from our readers and value their advice. Do you have any suggestions for improving Math Links? Topics for new articles? Drop us a line....either by E-mail or regular mail... to the attention of our Managing Editor. We enjoy hearing from you...!

TABLE OF CONTENTS:

WHAT IS MATHEMATICS?

1. Mathematics is non-existent but it makes us understand this existing phenomenal universe.

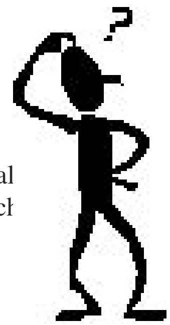
2. Mathematics is a human-made universe.

3. Mathematics is the cross-road of Arts and Sciences.

4. Mathematics is the Queen and Servant of Sciences.

5. Mathematics is a creative art.

6. Mathematics is just formalized common sense because it merely consists of formal consequences of self-evident truths, which are just part and parcel of our common intuition!



Your Morning Smile
smile@globeandmail.ca

Without geometry, life is pointless.
- T.J. (Thom) Park, Oshawa

VARIOUS POINTS OF VIEW ON THE CONCEPT OF A "POINT"

A point is a geometric entity having no length, width, or height.

A point is an extremity of a line segment.

A point is that which is indivisible.

A point is that with position only.

A point is an extremity which has no dimension.

A point is the indivisible beginning of all magnitudes.

These are not definitions of the concept of a 'point' in geometry but, rather, attempts to express what already one "knows" about points. But can one define the concept of a point?

MATHEMATICS AND .....??? At The University of Manitoba

G. Woods
University of Manitoba
Department of Mathematics

So, you say that you really like mathematics and you're pretty good at it, and you would like to take math at university..... BUT, there are other things that interest you too, and besides, you have to think about a career someday (at least they all keep telling you that). What can you do?

If you attend the University of Manitoba, there are lots of ways in which you can combine your enjoyment of math with your interest in other fields. Some of them you probably know about; others may come as news.

Let's start with a field that most people know about, namely Engineering. If you are technically inclined, and if you enjoy and are good at mathematics and physics, this may be the field for you. Engineering involves **lots** of mathematics. To oversimplify somewhat, electrical and computer engineering involve the most mathematics, civil engineering involves the least, and mechanical engineering is somewhere between.

A less widely known profession is actuarial science. Actuaries are the people who use mathematics and statistics to calculate what the premiums for insurance policies, pension plans, etc. should be. If you own a car, the size of your Autopac premium was worked out by actuaries, using data about past experience with claims, etc.. Winnipeg has a big insurance industry (Great West Life has its headquarters here) and actuaries are in demand (and well paid).

There are several programs at the University of Manitoba that involve actuarial science. If you are oriented towards the business world, you might want to enter the Faculty of Management and study actuarial science there. If you like the more theoretical aspects of the subject, you can take an Honours program in Actuarial Science, or Actuarial Science and Mathematics, or Actuarial Science and Statistics, within the Faculty of Science.

Perhaps you like physics, but you are more interested in the scientific ideas than in the practical applications (like Engineering). To oversimplify again, physicists come in two flavours, namely experimental physicists (who work in labs and undertake experiments to learn more about the universe) and theoretical physicists (who use mathematics to construct theoretical "models" to explain the experimental results). Albert Einstein was a theoretical physicist. If theoretical physics appeals to you, or if you want to learn in detail about how mathematics can be used in a scientific field, you might want to enter the Faculty of Science and take a Joint Honours program in Mathematics and Physics. It contains approximately equal numbers of courses from each field.

Lots of people who like mathematics also like working with computers. In forty years or so computer science has grown from almost nothing to a huge field with many different aspects. Some of them involve very little mathematics, but some of the more theoretical aspects are very mathematical indeed. Mathematically oriented computer scientists (or, mathematicians with an interest in computing

(continued on page 4)

(these things blur at the edges) study things like "computational complexity" (i.e. how intrinsically complicated a computing problem is) and the design of computer algorithms. Maybe you have heard of Alan Turing, who was (in essence) a theoretical computer scientist in the days before computers! He was a leader of the team (which included many mathematicians) of British code-breakers in World War II that cracked the code that the German military command used to communicate with its U-boats and air force. Their efforts were crucial in winning the war. Who says that mathematics isn't relevant?

"Scientific computing" has become an important discipline in recent years. Mathematicians use computers to work out approximate solutions to really complicated equations that arise in engineering, physical and biological science, and finance - equations that are too difficult to solve exactly.

To prepare for a career in areas that blend computer science and mathematics in this way, you should take Joint Honours Mathematics and Computer Science within the Faculty of Science. People with this expertise are very much in demand.

Statistics is a math-related field devoted to drawing conclusions about past or future behaviour by a mathematical analysis of data. (If this sounds a lot like actuarial science, don't be surprised. Actuarial science is more specialized than statistics, but it makes use of a lot of statistics). Statistics has both theoretical and practical components; if you are interested in the theoretical aspects of the subject, you can take Joint Honours Mathematics and Statistics within the Faculty of Science.

Mathematical economics is that branch of economics that constructs mathematical representations, or "models", of the economy and uses them, in conjunction with economic data, to make predictions about how the economy will behave in the future, and what the effects of different policy options would be on the economy. You can be sure that Alan Greenspan (Head of the U.S. Federal Reserve Board, and in effect the chief economist of the U.S. government) studied lots of mathematics. Since the "laws" of economics are not as precise as those of physics or actuarial science, mathematical economics is not as sharp a tool as (say) theoretical physics. Nonetheless, it is used a lot, and its practitioners can have rewarding careers. We have Joint Honours Mathematics and Economics programs on the books, should you be interested in this field.

A related and growing field is the mathematics of finance. Big institutional investors in the stock market (pension plans, etc.) use computer programs to trigger decisions to buy or sell huge blocks of stock depending on the market's behaviour. Behind these programs are mathematical "models" of how the market will work; these have been prepared by mathematicians who specialize in the world of finance.

Maybe you didn't know that mathematics now plays a central role in parts of the biological sciences. It is used to model the relationship between predator populations and prey populations (say, owls and mice) over time. Recently, a biologically-oriented mathematician in our department was part of a team of Canadian scientists that modelled the spread of the SARS epidemic that struck Toronto two years ago. More generally, mathematicians play a crucial role in studying how highly contagious diseases spread. These are just a few examples of how mathematics is used in biology.

All of this just scratches the surface of professions that make heavy use of mathematics. But can't you study math all by itself, just for fun? Do you have to be interested in using it to do so something else? Not at all - many people will study mathematics (in a major program in the Faculties of Arts or Science, or in an Honours Mathematics program within the Faculty of Science) for its own sake, because it is fascinating to them.

What happens to these people? A few become professors of mathematics at colleges and universities, and earn their living by teaching and doing research in mathematics. (Every year researchers discover new mathematical concepts and truths.) Very soon there will be lots of openings for qualified people who want to do this ("qualified" means possessing a Ph.D. in mathematics), because a great many of those who now teach mathematics at universities and colleges are about to retire.

More generally, specializing in mathematics at university can give you a way of looking at the world, and analyzing problems, that makes you valuable in many occupations. As an analogy, a highly conditioned athlete is in a position to become proficient at any one of a number of sports. A person who can write a clear, well organized persuasive essay will be an asset in a wide range of occupations that require good communication skills. Similarly, a person who understands university-level mathematics will be valuable in many technical and education related occupations.

For more information on math related careers and on choice of math courses in your first year at the University of Manitoba, go to our website at:

http://www.umanitoba.ca/science/mathematics

then, under "Undergraduate Information", click on each of: Undergraduate Programs, Undergraduate Courses, Choose your Math courses, and Careers in Mathematics.

I hope we will see some of you in our programs in future years!

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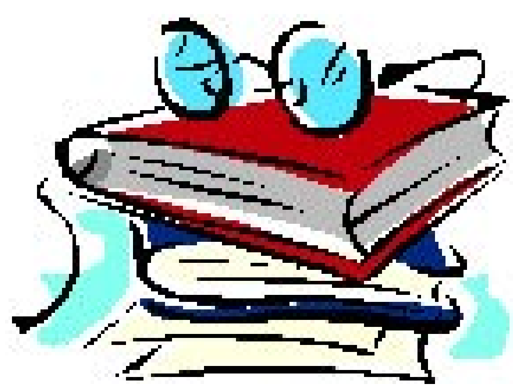

PARADOX IN MATHEMATICS

N.S. Mendelsohn
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Department of Mathematics

A paradox is a situation which appears in contradiction to our intuitive notions. Many paradoxes are merely jokes such as the one which appears in the Gilbert and Sullivan operetta where one of the characters muses over the fact that he has lived 28 years but has had only seven birthdays since he was born on February 29. Other paradoxes are more serious in nature such as the three paradoxes of Zeno. We will not pursue the matter of the Zeno paradoxes here, but if the rest of this article turns out to be of interest the student would be well advised to read up on these paradoxes. They do lead to an understanding of the notion of continuity in mathematics. In what follows we present two examples of paradoxes which arise from the analysis of baseball batting averages.

In baseball the batting average of a player is defined as the number of hits divided by the number of times at bat. Baseball afficionados love to break down the raw statistics into various sub classifications. For instance, it is well known that players perform differently when they bat against left-handed pitchers. Other situations arise when the players bat under the stress of a "clutch" situation. There is a wealth of statistics which are kept by all professional baseball teams.

We present two highly paradoxical examples.

Example 1.

In this example we compare the batting averages of two baseball players and break down the times at bat to two situations - times at bat against right-handed pitchers and against left-handed pitchers. The data appears in the following tables.

Table 1.

Player A	*Times at Bat*	*Hits*	*Av.*
Against right-handed pitchers	202	45	.223
Against left-handed pitchers	250	71	.284
Overall	452	116	.257

Player B	*Times at Bat*	*Hits*	*Av.*
Against right-handed pitchers	250	58	.232
Against left-handed pitchers	108	32	.296
Overall	358	90	.251

Observe that B has a better batting average than A, both against right-handed pitchers and against left-handed pitchers. But look, A's overall batting average is better than B's. Is there something wrong here? Yes, there is. What is wrong is our intuition about how averages behave. The mathematics and the answers are correct. This kind of paradox was first observed by E.H. Simpson in 1951. Mathematicians and statisticians now refer to it as Simpson's paradox. The particular example quoted here is due to Prof. T.H. Knapp of the University of Rochester. A point of interest is that this paradox occurs in real life examples although not very frequently. It probably has occurred in real life situations during a baseball season.

In the above example we have divided the batting situations into two cases. In fact, the paradox can occur by breaking down the times at bat into many types of situations. It is possible for B to be better than A for every one of the types of situations but overall A is better than B.

The author of this note has constructed an example in which the times at bat have been broken down into 16 categories. In each of these categores A is a better hitter than B but overall B is better than A (one of the categories is - batting against a right-hand pitcher with runners on first and third base). Can you guess the other 15 categories?

In our second example we consider a less complicated case - namely a breakdown of times at bat into four cases.

Example 2.

Statistics on a classification into four situations (see Table 2 below).

Table 2.

Sit. 1. No runner on base
Sit. 2. One runner on base
Sit. 3. Two runners on base
Sit. 4. Bases loaded.

Situation	1		2		3		4	
	A	B	A	B	A	B	A	B
Times at bat	104	156	60	88	112	70	90	138
Hits	28	40	21	30	25	15	30	45
Batting Avg.	.269	.256	.350	.341	.223	.215	.333	.326

In all situations A has a better batting average than B. But let us look at the overall statistics.

Table 3.

Batter	A	B
Times at bat	366	452
Hits	104	130
Batting average	.284	.288

Overall, B is a better hitter than A.

It is an interesting exercise to construct examples like these. The curious student will find it worthwhile to read some literature on weighted averages. It will give him a good deal of insight into how the paradox occurs.

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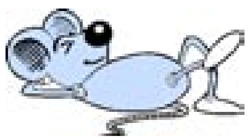
COOL WEBSITES TO EXPLORE



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Department of Mathematics

Science Fair Projects in Mathematics

Cool Math Games - 15 Puzzle
http://www.coolmath-games.com/15puzzle/index.html

Mathematics Is Everywhere
http://www.mathaware.org/mam/00/918/index.html

Polyhedra
http://mathforum.org/alejandre/workshops/unit14.html

Magic Squares
http://mathforum.org/alejandre/magic.square.html

Fractals
http://mathforum.org/alejandre/workshops/fractal/fractal3.html

A collection of ideas sorted by math topic, from the Canadian Mathematical Society.
http://camel.cecm.sfu.ca/Education/mpsf/
http://camel.math.ca/Education/mpsf/

A sample list of successful Canadian math projects
http://www.pims.math.ca/education/2000/regional/

Cool Math Projects (for ages 13-100)
http://www.coolmath.com/

Experiment-based math projects
http://www.juliantrubin.com/mathematicsprojects.html

The Seven Bridges of Königsberg heralds the beginning of graph theory and topology.
http://mathforum.org/isaac/problems/bridges1.html

Famous Problems in Math
http://mathforum.org/isaac/mathhist.html

Books:

[1] Howard Eves, *An Introduction to History of Mathematics* Holt, Rinehart and Winston, Toronto, 1986

[2] Gardner, Robert. *Science Projects about Math.* Carolina 1999 QA40.5.G374
Twenty-five projects and experiments that make use of mathematics in the context of science.

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SCIENCE FAIR PROJECTS IN MATHEMATICA

R. Padmanabhan
University of Manitoba
Department of Mathematics

Whenever we think of Science museums, we usually think of huge skeletons of dinosaurs, dancing of colorful laser beams, holograph paintings, adventures in space, etc. but never take a random walk through the primes! We do not see too many Mathematical displays simply because it is thought of as an 'abstract' science of ideas, and at best, only "dull" write-ups in the form of posters are the common math exhibits.

The fact is that most of the abstract mathematical concepts do arise from very concrete physical situations and hence it is certainly possible to design dynamic and interactive mathematical exhibits. Here, by 'interactive' we do not necessarily mean just pushing buttons but the ability of the exhibit to take along the mind of the audience to a conceptual tour of the area. The purpose of this section is to offer some guidelines as to the nature of such mathematical topics, deep results in that topic and a conceivable exhibit to convey the intricacy, beauty and the depth of those results.

My first suggestion for such a project is the "all-pervading" Mathematics itself! We can easily 'picture' mathematics in the form of a tree with roots, branches, leaves, fruits and so on. The analogy must be pretty obvious here: roots are the foundations or the original motivations or reasons; various branches are the 'different' areas of mathematics; the connecting links are the interrelations etc.. The fruits or 'gems' as I would like to picture them are the beautiful results in that area: for example

"$e^{\iota\pi} + 1 = 0$" and the 'prime number theorem'

are two such gems of mathematics.

You may not yet know these results and these are normally taught in a university math program.

The purpose of such a tree is to point out to the students as well as the visitors not only the historical growth of the subject but also the route it leads to other areas of knowledge and hence will guide the student to chalk out a career path.

You can see on the following page (page 7) a diagram of such a Tree of Mathematics.

The idea that it should look more like a Banyan tree (multiple trunks) rather than an Oak tree (single trunk) goes back to Howard Eves (CF.[1] p.489). Now a concrete display would be to take such a diagram and complete it, especially on the outer boundaries, so that it shows most of the known links to other areas of human endeavour like Physics, Chemistry, Biology, Engineering, Economics, Architecture and even Fine Arts etc. A list of websites describing various math projects is given in "Cool Websites". Good luck with your project!

[1]. Howard Eves, *An Introduction to the History of Mathematics*, Holt, Rinehart and Winston, Toronto, 1976.

PROBLEM CORNER

D. Trim
University of Manitoba
Department of Mathematics

Dear Readers:

Welcome once again to the **PROBLEM CORNER**. Here is the problem from the last column and its solution:

Problem:

> A four-digit number, which is a perfect square, is created by writing Anne's age in years followed by Tom's age in years. Similarly, in 31 years, their ages in the same order will again form a four-digit perfect square. Determine the present ages of Anne and Tom.

Solution:

> Their ages must be two-digit numbers, for if either is one-digit, the other must be three-digit, and when 31 is added to the one-digit age, the result is a two-digit number. This would contradict that in 31 years juxtaposing ages is a four-digit number. Let x and y be the two digit-ages of Anne and Tom. Then, $100x + y$ is a square, and we write that $100x + y = n^2$, where $n > 0$ is an integer. In 31 years, their ages are $x + 31$ and $y + 31$, and we can write that $100(x + 31) + (y + 31) = m^2$, where $m > 0$ is an integer. When we subtract these expressions:
> $100(x + 31) + (y + 31) - (100x + y) = 31{\cdot}101$
> $= m^2 - n^2 = (m + n)\,(m-n)$.

Since 31 and 101 are primes, it follows that $m + n = 101$ and $m - n = 31$, the solution of which is $m = 66$ and $n = 35$. Knowing that $100x + y = 35^2 = 1225$ and that x and y are each two digit numbers, it follows that $x = 12$ and $y = 25$.

I am sorry to report that I received no submissions on this problem. Let's blame it on the end of year and summer holidays. Perhaps you will be able to devote some time to the new problem.

Here is your new problem:

> Members of the math club at your school are either teenagers or twelve year olds. When their ages are multiplied together, the product is 163,762,560.
>
> How many members does the club have?
>
> Would it be possible for the product to remain the same if the club contained only teenagers?

Send submissions on this problem to:

S. Kangas, Managing Editor
Manitoba Math Links
Department of Mathematics
The University of Manitoba
Winnipeg, MB
R3T 2N2

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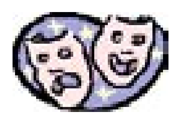

Four friends have been doing really well in their calculus class; they have received top grades for their homework and on the midterm. So when it's time for the final, they decide not to study on the weekend before, but to drive to another friend's birthday party in another city - even though the exam is scheduled for Monday morning. As it happens, they drink too much at the party, and on Monday morning, they are all hungover and oversleep. When they finally arrive on campus, the exam is already over.

They go to the professor's office and offer him an explanation, "We went to our friend's birthday party, and when we were driving back home very early on Monday morning, we suddenly had a flat tire. We had no spare, and since were driving on back roads, it took hours until we got help".

The professor nods sympathetically and says, "I see that it was not your fault. I will allow you to make up for the missed exam tomorrow morning."

When they arrive early on Tuesday morning, the students are taken by the professor to a large lecture hall and are seated so far apart from each other that, even if they were to try, they would have no chance to cheat. The exam booklets are already in place, and confidently, the students start writing. The first question - five points out of 100 - is a simple exercise in integration, and all four finish it within 10 minutes. The first to complete the problem turns over the page of the exam booklet and reads the next one: Problem 2 (95 points out of 100): *Which tire went flat?*

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