Volume IV, Issue #1, Fall 2003

UNIVERSITY
OF MANITOBA

MANITOBA MATH LINKS

BEAUTY PRINCIPLES DISCOVERY

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Summer Math Camp 2003

Alexander MacDonald
Camp Participant

Math camp was a memorable experience that I, and everyone else who attended, will never forget.One of the best aspects about the camp was the select group of people chosen to attend. With only 20 people it was easy to get to know everyone. At some other camps that I have attended where there were 200 people, I really only got to know 5 or 6 well.

Some things that were not so fun were that we were on the 9th floor of Speechly Hall during a heat wave, so none of us could sleep, and all our classes were in St. John's College so it was a full day's exercise just walking from our dorm to our first class. Speaking of exercise, right before dinner every day we had an outdoor exercise period where we played a sport (Monday was volleyball, Tuesday was soccer and Wednesday was too hot!) Other than those three flaws, things were exceedingly good. The schedule was good, the classes were good and the food was good.

We had two lectures a day and after them we had a problem solving session. That was more or less it, as far as the mathematical aspect is concerned. On Monday, we studied Euclidean Algorithm and Diophantine Equations. Euclidean Algorithm is used to find the GCF of large numbers and Diophantine Equations are used to find x and y (in separate equations) in terms of n. On Tuesday, we learned permutations and combinations, which are used in probability. On Wednesday, we learned about the pigeonhole principle and about the Ramsay Theory. I won't get into that for I am currently too busy trying to calculate $R(5,5)$. Thursday, we learned more on probability but more importantly I learned that Dr.Trim was right about probability, it isn't particularly exciting. That evening we had a speaker come in and tell us about *infigers*. I think I had better explain this because it defies everything I have ever learned in math. For the sake of ease *infigers* are a different way of counting which is similar to the decimal system but it deals with repeating numbers differently. Here is the best explanation I can give. If I asked you what "1/3 is in decimal, chances are that you would say 0.3333. In *infigers* it is ...667; here is the rationale behind it. If you multiplied 1/3 by 3 you would get 1 so if you multiplied ...667 by 3 you should also get 1. If you multiplied ...667 by 3 you get ...001, which is close to 1. It was pointed out that you are carrying a 2 in each digit of the calculation. Because the 2 is being pushed over an infinite number of times, it falls off the end of the earth. If you are confused let me tell you, you are not alone.

All in all, I enjoyed myself and would go back in a heartbeat. I was mentally challenged, to some extent physically challenged, and I was among people who shared at least one thing with me, the love of Mathematical Science.

****

Back : L to R: Dr.D.Trim, N.Harland, J.Huska, J.Taylor, A.MacDonald, T.Cao, T.Ting, T.Cao, R.Tetlock, S.Lee, L.Horosko, A.Nguyen, E.Himbeault, D.Lee.
Front : L to R: A.Duca, S.Abdulrehmaa K.Dhillon, C.Siddall, C.Simons, W. Korytowski. *Missing:Z.He, K. Liu, S.Xu, J. Zhang*

Manitoba Math Links is financially supported by The Winnipeg Foundation

MANITOBA MATH LINKS WEB SITE
........www.umanitoba.ca/science/mathematics
(a PDF Version of Math Links is available on this website)

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MANITOBA MATH LINKS

The Math Links Newsletter is published by the Mathematics Department Outreach Committee three times a year (Fall, Winter & Spring).

CAREER PROFILES IN MATHEMATICS

The following web sites contain short descriptions of jobs held by individuals with mathematics degrees and a wide variety of backgrounds:
http://www.siam.org/careers/career2.htm#numberfive
http://www.maa.org/careers/index.html

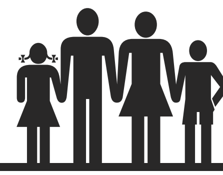
A NOTE FROM THE EDITORS:

We welcome comments from our readers and value their advice. Do you have any suggestions for improving Math Links? Topics for new articles? Drop us a line....either by E-mail or regular mail... to the attention of our Managing Editor. We enjoy hearing from you...!

IMPORTANT DATES TO REMEMBER:

Mathematics Problem Solving Workshop 2004

Jan 24 & 31
Feb 7 & 14



SPEAKERS AVAILABLE

If you are interested in having a faculty member come to your school and speak to students, please contact our Managing Editor at *kangass@cc.umanitoba.ca* or phone 474-8703.

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A-Mazeing Mathematics: Finding Your Way Through a Maze

P. W. Aitchison
Mathematics Dept

Everybody has probably heard of some of the famous European mazes, such as the Hampton Court Maze or the Dublin Maze that have to be navigated through a confusing arrangement of hedges and openings to the centre of the maze. There is also a history of mazes in North America including the Hopi Indian circular mazes. In summer a number of farmers around Winnipeg create mazes in their fields using corn plants instead of hedges. One of these that is open to the public is found on St. Mary's road south of the perimeter highway. A good introduction on the internet is "Jo Edkin's Maze Page" at http://www.gwydir.demon.co.uk/jo/maze/branch.htm.

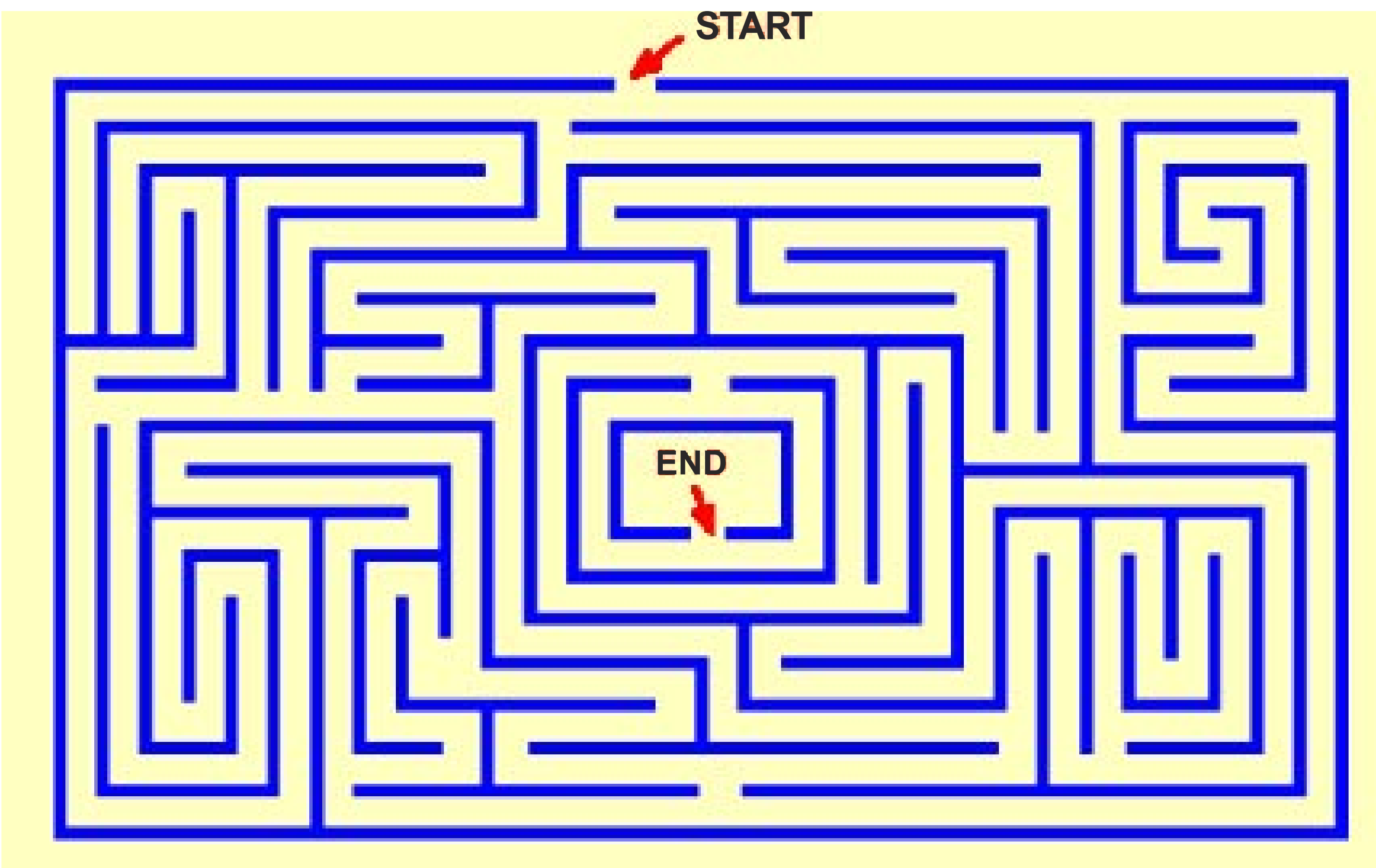
This article is about using mathematics to navigate mazes. A typical maze is generally viewed as a two-dimensional flat area with walls/ hedges making walkways between the walls and occasional openings into other walkways and sometimes more complicated junctions where there are lots of possible choices. This kind of maze can be drawn on a piece of paper showing, more or less, how it would be viewed by a bird looking down on the maze. A typical maze of this type in the picture below is from Warren Street Underground Station in London, England. It was created to occupy the time of waiting passengers and was designed to take longer than the typical time of three minutes between trains. Try it for yourself!

However, mazes do not have to be two-dimensional and there is no reason that they should be except that it is much more difficult to construct three-dimensional mazes. In fact there are problems that we can encounter that are equivalent to the problem of traversing a three dimensional maze. For example, suppose that you have to explore a large system of caves that has just been found and is not yet explored. Cave systems often have passageways that branch and rejoin in incredibly complicated patterns just like a maze. Similarly, suppose that you have to search a very large building with an irregular arrangement of passageways; then how would you do that without missing any of it and preferably as efficiently as possible?

THE PROBLEM WE WANT YOU TO ANSWER:
How can we reliably find our way through a two or three-dimensional maze without a map of the maze, or search the whole of a complicated cave system or building? Secondly, how can we do that in the most efficient way? Try to come up with an answer of your own to this problem before my solution is published in the next newsletter. The rest of this article gives some ideas on this problem.

What can we do with mathematics? At first sight it is not obvious that we can do anything, but on second thought it is a problem of analyzing a complicated and well-defined situation and that is what mathematics is all about. Notice that this problem has some very peculiar features. We are assuming that we are confronted by a maze that we do not know anything about and we definitely do not have a picture of it like the Warren Street Maze shown . That means we have to find a way of searching reliably this system of passageways without knowing anything about it at all! We do not know how big it is, or how many passageways and junctions there are, or even if it is possible to reach the place we want to go to! Clearly there will be no formula that gives the answer in advance of which way to go at each junction. Instead what we want to devise is a process or algorithm for searching the maze completely without missing any of it. We have to be able to search every piece of a maze because we cannot know in advance where the end is, and it could be on the very last piece that we search.

(continued on page 4)



Let's look at some aspects of the solution, but without attempting to give a complete answer at this time:

1. The answer to the question has to be reasonably simple and easy to use.

2. Every time we come to a junction we can only take one of the choices of passageways. Consequently assume there is some simple method of marking the passageways that we used to enter and leave the junction, so that when we come back to that junction to search other passageways we will know what directions were taken previously. We are assuming here that you are not using a ball of string to keep track of your trail and do not have an incredible photographic memory!

3. We will need a standard procedure to deal with a number of different situations:

    a. What action do you take when you come to a junction that has never been visited before? That is, none of the passageways have any of the visiting marks.

    b. What action do you take when you come to a junction that has been visited before ? (Be careful about your answer to this one because it is very easy to irrevocably damage your search if you make the wrong decision.)

    c. What do you do when you come to a dead-end? Well maybe that is obvious, but the algorithm should explicitly say what to do.

    d. How can you tell if the maze is not possible? That is, there is no connection to the centre of the maze. The algorithm needs to be able to deal with this situation.

4. Clearly some passageways will generally be traversed more than once, since there is no choice if you reach a dead-end. Decide whether or not any passageways may have to be traversed more than two times. What would be the "worst case" search with the greatest possible length of travel of the passageways?

5. For an ecologically-guided approach to this problem we might want to minimize the number of passage markers used. In one version of this problem the only allowed markers are some small stones or pebbles. Think about what would be the minimum number possible of pebbles needed to solve the worst case search. Mathematicians would call this the minimax answer of minimizing the maximum (the worst case). The answer will be different for two and three-dimensional mazes.

Send your answer to :

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The University of Manitoba
Winnipeg, MB R3T 2N2

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Graphs: Cutting and Pasting

Roy & Diane Dowling
Mathematics Department

Mathematics texts tell us how to write down the equations of some geometric figures such as straight lines, circles, and parabolas. It is challenging to try to find equations for some other familiar shapes. Think of the shapes of numerical digits and letters of the alphabet. Can we write equations for some of them? Suppose we look at the figure "8". This figure can be made by pasting one circle above another. It is easy to write down an equation for each of these two circles but can we find an equation for the figure obtained by pasting the two circles together? Actually, it is quite easy to do so. Next, look at the letter C. This letter could be made by taking a circle and cutting off part of the right side. It is not a problem to write down an equation for a circle. It turns out that it is also possible to write down an equation for the figure obtained by cutting off a part of that circle.

First we will see how we can find an equation for a graph which has the shape of the figure "8". By the union of two graphs we mean the graph consisting of all points which lie on one of the graphs or on both of them.

The graph of: $x^2 + (y-1)^2 - 1 = 0$ ... (1)
is a circle of radius 1 with centre at (0,1).

The graph of : $x^2 + (y+1)^2 - 1 = 0$ ... (2)
is a circle of radius 1 with centre at (0,-1).

The union of these two graphs is the figure "8" graph shown in Diagram 1

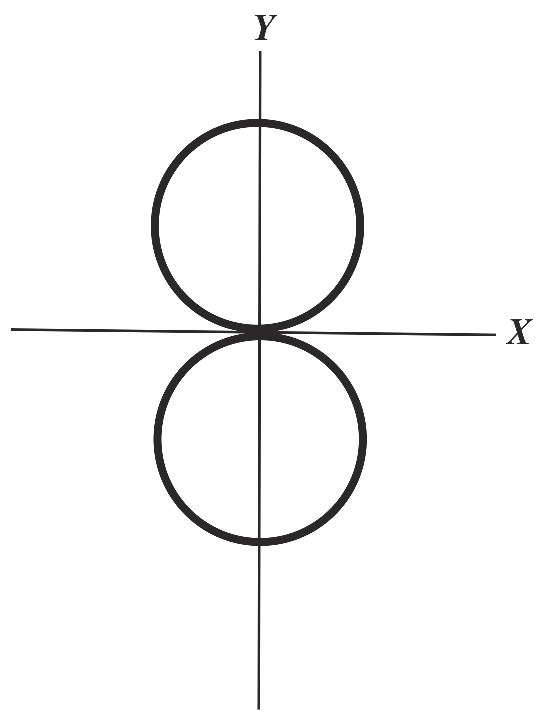

Diagram 1

An equation for this union is:
$(x^2 + (y+1)^2 - 1)(x^2 + (y-1)^2 - 1) = 0$ ...(3)

To see why this is true we note that the equation (3) holds if, and only if, one or both of the equations (1) and (2) hold and so the graph of (3) is the union of the graphs of (1) and (2). Thus the graph of $(x^2 + (y+1)^2 - 1)(x^2 + (y-1)^2 - 1) = 0$ is the graph shown in Diagram 1.

Having talked about the union of two graphs let's look at the possibility of finding the equation of the intersection of two graphs whose equations are known. By the intersection of two graphs we mean the graph consisting of all those points which are common to both of the graphs. The graph of the equation $y - x = 0$ is the straight line shown in Diagram 2 and the graph of the equation $y - |x| = 0$ is shown in Diagram 3. The intersection of these two graphs consists of all the points common to these two graphs and is therefore the infinitely long half-line starting at the origin shown in Diagram 4.

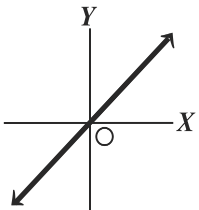

Diagram 2

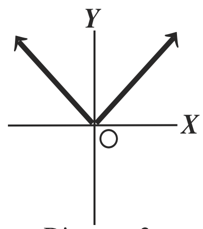

Diagram 3

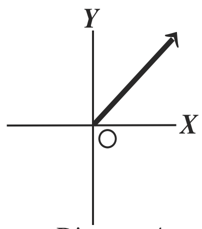

Diagram 4

An equation for this intersection is $(y - x)^2 + (y - |x|)^2 = 0$. Here is the reasoning: x and y are real numbers so this equation is true if, and only if, ***both*** of the equations $y - x = 0$ and $y - |x| = 0$ are true. It follows that $(y - x)^2 + (y - |x|)^2 = 0$ is an equation for the intersection of the graph of the equation $y - x = 0$ and the graph of the equation $y - |x| = 0$ and so it is the equation of the graph shown in Diagram 4.

Soon we will see how we can write down an equation for a graph having the shape of a letter C but first we will look at an equation whose graph is really humongous. We start by looking at the expression $x + |x|$ where x is a real number. This expression has the value zero if and only if x is negative or zero. It follows that the graph of the equation $x + |x| = 0$ consists of all those points in the plane whose x-value is negative or zero and so is the infinite region consisting of all points on and to the left of the y-axis as shown in Diagram 5.

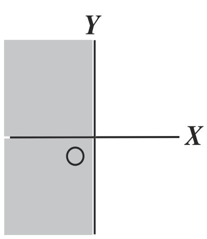

Diagram 5

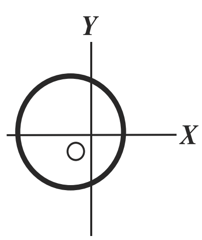

Diagram 6

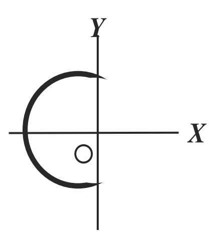

Diagram 7

To find an equation for a graph shaped like the letter *C*, we will first consider the circle with centre at (-1,0) and radius 2. Its equation is $(x+1)^2 + y^2 - 4 = 0$ and its graph is shown in Diagram 6. If we cut from this graph the part to the right of the y-axis we will get a figure shaped like the letter *C*. Now consider the equation:

$$\{(x+1)^2 + y^2 - 4\}^2 + \{x + |x|\}^2 = 0.$$

The graph of this equation is the intersection of the graph of $(x+1)^2 + y^2 - 4 = 0$ and the graph of $x + |x| = 0$ and so consists of those points on the circle in Diagram 6 which also lie on the graph shown in Diagram 5. Thus it is the figure shaped like the letter *C* shown in Diagram 7.

For finding equations of graphs in the shape of some letters and numerical digits it is handy to be able to write an equation for a line segment. As an example we might consider the line segment joining the points (-1,-1) to (1,1). This line segment can be obtained by cutting the infinite line with equation $y - x = 0$ at the points (-1,-1) and (1,1) and discarding the unwanted parts. Before describing how to write an equation of this line segment we will first look at another equation and its graph.

For a point (x,y) the perpendicular distance to the line whose equation is $x = -1$ is $|x+1|$ and the perpendicular distance to the line whose equation is $x = 1$ is $|x-1|$. The perpendicular distance between these two lines is 2. It follows that the equation $|x+1| + |x-1| = 2$ is satisfied if and only if the point (x,y) is between these two lines or lies on one of them. The graph of the equation $|x+1| + |x-1| = 2$ therefore consists of all those points which are between or on the lines whose equations are $x = -1$ and $x = 1$. (See Diagram 8.)

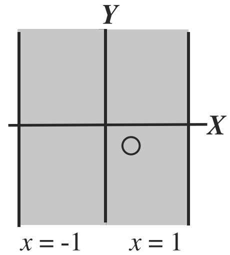

Diagram 8

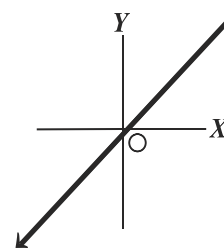

Diagram 9

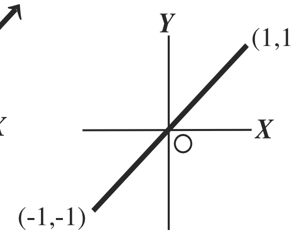

Diagram 10

We can now see how to draw the graph of the equation

$$(x - y)^2 + (|x + 1| - |x - 1| - 2)^2 = 0.$$

It is the intersection of the graph of $x - y = 0$ shown in Diagram 9 and the graph of $|x+1| - |x - 1| - 2 = 0$. So it consists of those points on the line shown in Diagram 9 which also lie on the shaded graph shown in Diagram 8. In other words it is the line segment joining (-1,-1) and (1,1) as shown in Diagram 10.

The examples given might help you provide answers to the following quiz:

for each of the following, name the letter of the alphabet or digit between 1 and 9 which is shaped like the graph of the given equation.

(a) $(x^2 - y^2)^2 + (|x + 1| - |x - 1| - 2)^2 = 0.$
(b) $\{(x - y)(y - 1)\}^2 + \{|x + 1| - |x - 1| - 2\}^2 = 0.$
(c) $\{(x+1)^2 + y^2 - 4)(y + 2)\}^2 + \{|x + 1| - |x - 1| - 2\}^2 = 0.$
(d) $x^2 + (|y + 1| - |y - 1| - 2)^2 = 0.$
(e) $\{(y+1-x)(y+1+x)\}^2 + \{|y + 1| - |y - 1| - 2\}^2 = 0$
(f) $\{(y-1)(y+1)(y-x)\}^2 + \{|x + 1| - |x - 1| - 2\}^2 = 0.$

Answers:
(a) X; (b) Y; (c) 2; (d) I; (e) V; (f) Z.

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A MATH LIMERICK

A mathematician confided
That a mobius band is one-sided
And you'll get quite a laugh
If you cut one in half
For it stays in one piece when divided!

U of M Places 23rd In The Putnam!

R. Craigen
Mathematics Department

The William Lowell Putnam Competition is reputed to be the most difficult mathematics competition in the world. Each December 3000 university math students in North America write this exam of 12 questions in 6 hours. Results are announced in March (it takes that long to grade them!). In recent years the median score has generally been 0 or 1 (out of a possible 120!). An aggregate "team" score is used to rank each university among the 400 or so that participate. A top 100 ranking is considered to be a sign of excellence.

As coach of the U of M team since 1999, I have aimed to establish our rank permanently in the top 50. In those four years we have attained ranks of 80th, 121st, then 69th and, last year, 23rd! We also write a regional competition involving 70 teams of 3 students from colleges and universities in the North Central U.S., Saskatchewan, Northern Ontario and Manitoba. In each of the past 4 years a U of M team has placed in the top 4; last fall 2 of our teams tied for second place! Students who do well in these competitions are assured of offers from the best graduate schools, scholarships and recognition, but their most basic motivation is a desire to excel in a challenging and rewarding form of competition.

Most high school mathematics competitions are multiple-choice, whereas university-level contests require full worked solutions to open-ended problems. Students must be familiar with some standard problem solving tools and techniques, but most of our problems require no math beyond the high school level -- the key to success is not knowledge, but ingenuity. Open format questions are also seen in some high school contests, such as the Manitoba Mathematics Competition (whose problems are somewhat more accessible than the Putnam!). The Canadian Open Mathematics Challenge and the new Fryer, Galois and Hypatia exams for grades 9-11 are similar. The Canadian and International Mathematical Olympiads (CMO & IMO), written by invitation only, are like the Putnam exam in form and approach it in difficulty. If you are an aspiring mathlete hoping for an invitation to write these contests, you should be training now.

How does one train for this level of mathletics? If no group meets for this in your school here are some options: form your own group and ask a teacher to sponsor it. Subscribe to the (free) Olympiad Correspondence Program, run by the Canadian Mathematics Society; the web page for this program, *http://www.cms.math.ca/Competitions/MOCP/info.html,* also links to the International Mathematical Talent Search and to Crux Mathematicorum, a problem solving journal full of challenging problems at all levels and articles about mathletics. Also visit *http://server.maths.umanitoba.ca/~craigen/manitobamathletics*, a page I maintain for Manitoba mathletes and schools. Each Spring the Institute of Industrial Mathematical Sciences and our Department sponsors a 4 week problem solving workshop for high school students. We also run a summer math camp for mathletes in grades 9 and 10, part of a nation-wide training program.

A challenge for high school students aspiring to compete at the very highest levels: sit in on our team's training sessions (in addition to your school's). We meet Tuesdays, 4-5:30 PM to consider problems whose difficulty starts just below that of the CMO, so talented high school students should find many of them "doable". If you are interested in this, E-mail me at *craigenr@cc.umanitoba.ca*. These sessions should help you prepare for the CMO and IMO, and also should provide early preparation for undergraduate competitions. Will you be on our Putnam team when we break into the top 10 in the next few years?

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Twenty Questions

R.G. Woods
Mathematics Department

Fifty years ago there was a popular radio quiz show called "Twenty Questions". Listeners would suggest a common "mystery object" to the moderator of the show. A panel of "experts" was told by the moderator that the object was either "animal, vegetable or mineral". The panelists then asked the moderator a series of questions about the object, with the goal of guessing what the object was. Each question had to be answerable with either a "yes" or a "no". So, you could ask "Is the object in Canada?", but not "In which country is the object?".

As the name suggests, the panelists had 20 questions in which to guess the identity of the object. With all the different objects in the world, you might think that the panelists would seldom be successful. However, they guessed the object correctly a surprising large fraction of the time. This was partly because the panelists had a keen sense of what sorts of objects the "radio audience" would suggest to the moderator. However, another reason is that you can discover a lot more information with 20 well chosen questions than our intuition might suggest. The mathematics behind this isn't very deep, but it is perhaps rather counter-intuitive. Here it is.

We illustrate the situation with a simplified version of the game. Your friend chooses a positive integer no bigger than 500,000, and offers you a wager. You are allowed to ask her 20 questions, each capable of being answered by a "Yes" or "No". (Henceforth the word "question" will mean this sort of question.) If you guess the number correctly after asking 20 (or fewer) questions, you win. Otherwise she does. Should you take the bet?

At first sight it seems a poor bet for you to take. After all, there are 500,000 possible numbers, and you can't ask about each of them with only 20 questions. But perhaps there is a strategy that works. Before you continue reading, try to think of one.

Our strategy involves logarithms to the base 2, so we review these. Suppose that $k > 1$ and $n > 0$. Recall that "$n = \log_k y$" means that "$k^n = y$". We call $\log_k y$ the "logarithm of y to the base k". If k is fixed and y gets bigger, then $\log_k y$ gets bigger. Thus $\log_2 8 = 3$ (because $2^3 = 8$) and $\log_2 10$ is some number between 3 and 4 (= $\log_2 16$).

You can compute logarithms to any base on your scientific calculator. The "*ln*" key calculates logarithms to the base e ($e \cong 2.718$) and the "log" key calculates logarithms to the base 10. We will want logarithms to the base 2. You can calculate these using the "*ln*" key as follows. Suppose $n = \log_2 y$. Then $2^n = y$. So $ln\,(2^n) = ln\,(y)$. But $ln\,(2^n) = n\,ln\,(2)$ (using a well known property of logarithms). So:

$$\log_2(y) = n = (ln\,(y))/ln(2),$$

and we can express logarithms to the base 2 using only the *ln* key.

Our strategy is based on the following

Fact: If y is a positive integer between 1 and 2^k inclusive, then we can determine its value by asking no more than k questions.

To see this, note that we start by knowing that there are 2^k possible values for y. Our first question is: *Is* $1 \le y \le 2^{k-1}$*?* If the answer is "yes" then there are now 2^{k-1} possible values for y. If the answer is "no", then $2^{k-1} + 1 \le y \le 2^k$, so again there are now 2^{k-1} possible values for y. Our second question is: *Is y in the lower half of the latest possible range of values for it?* As our new possible range of values has size 2^{k-1}, the answer to the second question determines that y is in a range of values of size $2^{k-1} / 2 = 2^{k-2}$.

If we continue in this way, we see that after n questions, we know that y is in a range of values of size 2^{k-n}. So when n = k, we know that y is in a range of values of size $2^{k-k} = 2^0 = 1$; in other words, we now know what the value of y is. So: if $1 \le y \le 2^k$, we need at most k questions to obtain its value.

Note that if $y > 2^k$, we may need more than k questions. For example, if we know that y is an integer between 1 and 10 inclusive, we cannot necessarily determine its value with only 3 questions. Try it and see!

For this reason, if we are told that y is an integer between 1 and j, and if n is the unique integer for which $2^n < y \le 2^{n+1}$, we will act as though we were searching for an integer between 1 and 2^{n+1}, and proceed as described above. We will discuss below how to determine n.

To illustrate this with small numbers, suppose that we are asked to determine a number between 1 and 100 inclusive. Let's suppose that the number is 40 (although we don't know that when we start). As $2^6 = 64 < 100 < 128 = 2^7$, we have $n = 6$. Hence we expect to use 7 questions to find the unknown number. Since we might need more than 6 questions (as $100 > 2^6$), it doesn't increase the number of questions that we will need if we assume that the number that we are looking for lies between 1 and 128, rather than between 1 and 100. Here is how the questions/answers would go:

Question 1: Is y between 1 and 64? [Answer: yes]

Question 2: Is y between 1 and 32? [Answer: no] (So $33 \le y \le 64$*; the "lower half" of this is from 33 to 48)*

Question 3: Is y between 33 and 48? [Answer: yes]

Question 4: Is y between 33 and 40? [Answer: yes]

Question 5: Is y between 33 and 36? [Answer: no] (So $37 \le y \le 40$*)*

Question 6: Is y 37 or 38? [Answer: no] (So y is 39 or 40)

Question 7: Is $y = 39$*? [Answer: no]*

So after 7 questions, we have discovered that $y = 40$.

Now let's go back to our original question - should we accept the challenge to guess an integer between 1 and 500,000 in no more than 20 questions? From the discussion above, we need to find the smallest integer k for which $500{,}000 \le 2^k$. Armed with our trusty calculator, we compute:

$\log_2 (500{,}000) = (ln(500{,}000)) / ln(2) \cong 13.1224/0.6931 \cong 18.93$.

So $500{,}000 \cong 2^{18.93} < 2^{19}$. So we know that if we use the strategy described above, we can determine the number with no more 19 questions. You should take the bet! The general formula is this. You can determine a positive integer no larger than p using no more than $[\log_2(p)] + 1$ questions, where $[x]$ denotes the integer part of the number x. (Thus $[3.12] = 3$ and $[13] = 13$, for example.) If $\log_2(p)$ is an integer, you only need $\log_2(p)$ questions.

Another way of determining the integer y, given that $1 \le y \le 2^k$, is to determine its binary representation. From this you can deduce the value of y. We usually give the "base 10" representation of an integer; thus $5403 = 3(10^0) + 0(10^1) + 4(10^2) + 5(10^3)$. Similarly, each non-negative integer m can be written in a unique way as a sum of multiples of distinct powers of 2 (starting with 2^0 and working up), where each coefficient is either 0 or 1. If $0 \le m < 2^k$ then this representation need have no more than k digits. For example, $0 \le 39 < 2^7$, so we can write :

$$39 = d_0 2^0 + d_1 2^1 + d_2 2^2 + d_3 2^3 + d_4 2^4 + d_5 2^5 + d_6 2^6,$$

where each d_i is either 0 or 1. In fact, $d_0 = d_1 = d_2 = d_5 = 1$ and $d_3 = d_4 = d_6 = 0$, because
$39 = 1 + 2 + 4 + 32 = 2^0 + 2^1 + 2^2 + 2^5$.

The representation of 39 in binary notation (to 7 digits) is thus 0100111.

If I know that $1 \le y \le 2^7$, then $0 \le y - 1 < 2^7$, so I can represent y -1 using no more than 7 binary digits. In 7 questions I can determine the value of each digit, and thus the value of y -1 and hence y.

Are these two approaches related? Yes! In the example presented earlier where y was 40, we have $y - 1 = 39$, and its sequence of binary digits (left to right) can be determined by the answers to our sequence of questions when we determined that the unknown number was 40; if the n^{th} answer was "yes" the n^{th} digit is "0"; otherwise it is "1". This relation between the sequence of answers about y, and the sequence of binary digits of y-1, holds in general. (Why?)

Now $2^{20} = 1{,}048{,}576$. So, if there are no more than 1,048,576 objects available for guessing in a game of Twenty Questions, you can (in principle) determine the object. In practice, you must, at each stage of the game, choose a question that will cut the list of possible objects into two subsets of approximately equal size. The ability to do that is what distinguishes the Twenty Questions "expert" from the beginner, and keeps the game from being purely an exercise in mathematics.

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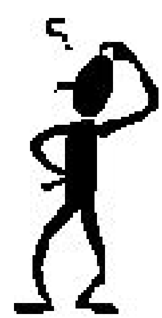

PROBLEM CORNER

D. Trim
Mathematics Dept

Dear Readers:

Welcome once again to PROBLEM CORNER. Here is the problem from the last column and its solution.

> ***Show that in any set of seven different positive integers, there will always be at least two of them whose squares differ by a multiple of 10.***

All integers end in one of the digits 0,1,2,3,4,5,6,7,8, or 9. When such numbers are squared, the last digit is 0,1,4,9,6,5,6,9,4, and 1, respectively. Notice that there are only six different digits, 0,1,4,5,6, and 9. It follows then that if seven integers are squared, at least two of them must end in the same digit. (This is an example of what is called the Pigeon Hole principle.) When two of these are subtracted the last digit is zero; that is, their difference is divisible by10.

I regret to inform you that I received no submissions on this problem. Send submissions on the next problem to:

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Here is the problem: ***If $n + 1$ integers are chosen from the first $2n$ positive integers {1,2,...,2n}, at least one of them must divide another. I will give you a hint: Use the Pigeon Hole principle and express each integer in terms of powers of 2.***

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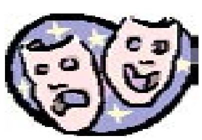


A math student and a computer science student are jogging together in a park when they hear a voice: "Please, help me!"

They stop and look. The voice belongs to a frog sitting in the grass.

"Please, help me!" the frog repeats. "I'm not really a frog; I'm an enchanted, handsome prince. Kiss me, and the spell will be broken: I will be yours forever..."

The computer science student picks up the frog. She examines it carefully from all sides - not even making an attempt to kiss it.

"You don't have to marry me", the frog continues frantically, "if you're afraid of the commitment. I'll do whatever you wish me to do if you'll just kiss me..."

The frog's voice is silenced when the computer science student puts the animal into her pocket.

"But why don't you kiss him?!" the math student asks.

"You know," she replies, "I simply don't have time for a boyfriend - but a frog that talks makes a really cool pet..."

Cool Websites

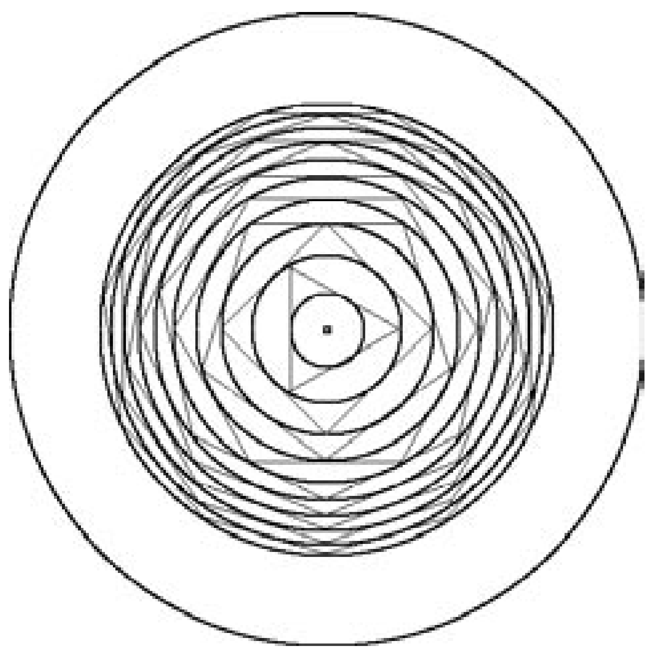
R. Padmanabhan
Mathematics Dept

Infinitely Exploding Circles

The circumcircle of a set of points is the smallest circle that completely encloses the set. To construct the all-embracing circumcircle of an equilateral triangle (i.e. a regular polygon with 3 sides) use the following algorithm:

1. construct it's circumcircle and enclose it in the smallest regular polygon with 4 sides (i.e. a square);
2. at each step, repeat the above step increasing the number of sides of the regular polygon by one;
3. continue indefinitely.

The radius of the outermost circumcircle grows quickly in the first few steps, but as the outermost polygons come closer and closer to a circle this growth slows to an ever more sluggish crawl and then virtually stops. Surprisingly, instead of continuing to expand, the radius approaches a maximum value, as illustrated in the drawing below. The mathematician and computer researcher Clifford A. Pickover discussed this construction in his fascinating book mentioned below. He explained that its radius is given by the reciprocal of the product $(\cos \pi/3) \times (\cos \pi/4) \times (\cos \pi/5) \times \ldots \times (\cos \pi/n)$ where n is the number of sides in the outer polygon. He programmed a computer to begin this tedious calculation and found that the first thousand polygons bring that radius only to 8.657231; even 10,000 iterations come no closer than 8.695745 whereas the actual ultimate value is 8.7000366252081945.. Draw it yourself and find the actual value you obtain.



For more details, visit the web sites mentioned below:
http://www.research.att.com/cgi-bin/access.cgi/as/njas/sequences/eisA.cgi?Anum=A051762
http://mathworld.wolfram.com/PolygonCircumscribing.html
http://www.recoveredscience.com/const306outerlimit.htm
Pickover, C. A. "*Infinitely Exploding Circles*." Ch. 18 in Keys to Infinity. New York: W. H. Freeman, 1995.
Curnow, Tamara, "Falling Down a Polygonal Well," *Mathematical Spectrum*, 26, 1993/1994,

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