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MANITOBA MATH LINKS

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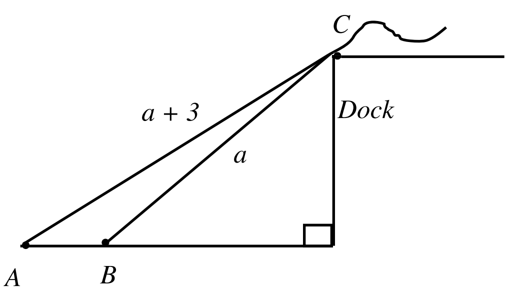
Two Problems With Nice Solutions

Diane and Roy Dowling
Mathematics Dept

Sometimes a young person with little experience in mathematics will solve a problem in a very clever way which might not occur to a more seasoned problem solver. A friend of ours in another province has been giving problem-solving workshops in high schools. At one of these workshops he presented the following problem:

A man stands at the edge of a dock holding a rope whose far end is attached to a boat which is lower than the level of the dock. He draws in some rope keeping it taut so that the distance between his hand and the boat is reduced by three metres. The boat moves a certain distance through the water in a straight line. Is this distance more or less than three metres?

Here is one person's solution to the problem:



In the diagram *A* is the initial position of the boat, *B* is the final position and *C* is at the man's hand. Let $BC = a$. Then $AC = a+ 3$. (We are to determine whether *AB* is less than or greater than 3.)
By the cosine law for triangles,

$$(AB)^2=(AC)^2+(BC)^2-2(AC)(BC)\cos C$$
$$= (a + 3)^2 +a^2 - 2 (a+3) (a) \cos C$$
$$= 2a^2 + 6a - (2a^2 + 6a) \cos C + 9$$
$$= (2a^2 + 6a) (1 - \cos C) + 9$$

Since $\cos C < 1$, $1 - \cos C$ is a positive number. So is $2a^2 +6a$. . Thus $(AB)^2$ is the sum of 9 and a positive number, so $(AB)^2 > 9$ and $AB>3$.

Most workshop participants came up with solutions which were at least as lengthy as this one. However, one young student said that it was unnecessary to do all that calculation and then presented a solution that was short and could be understood by someone in elementary school. Can *you* find a short solution to this problem? To see the nice solution given by the workshop participant see page 3.

Recently we came across a problem whose solution seemed to require a knowledge of advanced mathematics. To our surprise the solution was very elementary. To understand the problem you need to know what is meant by an irrational number. A real number such as $\frac{2}{3}$ or $\frac{9}{5}$ which can be written as the quotient of two integers is called a rational number. Some real numbers are not rational. For example, it is known that $\sqrt{2}$ is not rational. (For a proof go to the web site: http://www.math.utah.edu/~alfeld/math/q1.html)

Real numbers which are not rational are referred to as irrational. It is easy to see that a rational power of a rational number can be irrational. For example $2^{½}$ is the same as $\sqrt{2}$ and so is an irrational number. Here is the problem:

> Show that it is possible for an irrational power of an irrational number to be rational.

Have a try at solving the above problem. For a solution see page 3.

Thanks To The Winnipeg Foundation!!

The Department of Mathematics has received a grant from the Winnipeg Foundation to help with the production of Math Links. The money has been used to obtain a better printer (and associated software) for production of the newsletter. Funds will also be used to increase the number of copies being sent out to schools.

We very much appreciate this assistance and vote of confidence from the Winnipeg Foundation.

Dr. R. G. Woods, Head
Department of Mathematics

MANITOBA MATH LINKS WEBSITE
........www.umanitoba.ca/faculties/science/mathematics

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MANITOBA MATH LINKS

The Math Links Newsletter is published by the Mathematics Department Outreach Committee three times a year (Fall, Winter & Spring).

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CONTEST RESULTS

As no submissions were received........we have no results to announce......Y. Nought.

A NOTE FROM THE EDITORS:

We welcome comments from our readers ad value their advice. Do you have any suggestions for improving Math Links? Topics for new articles? Drop us a line....either by E-mail or regular mail... to the attention of our Newsletter Co-ordinator. We enjoy hearing from you...!

DATES TO MARK ON YOUR CALENDAR:

Problem Solving Workshop

Jan 25, Feb 1, 8 & 15, 2003

SPEAKERS AVAILABLE

If you are interested in having a faculty member come to your school and speak to students, please contact our Newsletter Co-ordinator.

TABLE OF CONTENTS:

A Week At MathCamp

Mandy Einarson
Camp Participant

Sunday, August 18, 2002.....with a nervous feeling in my stomach (like I always have when I'm about to meet new people), I grabbed my two bags from the backseat of my parents' truck and started across the parking lot for St. John's College. I had come for the University of Manitoba Math Camp and was about to register. As I approached the doors to St. John's College, I realized I had absolutely no reason to be nervous. I knew that over the next week I was going to learn many new (and interesting) things, along with meeting some really fantastic people. With the nervous feeling gone, I opened the door and started inside.

My name is Mandy Einarson and from August 18 to 23, I attended the University of Manitoba Math Camp at St. John's College. While at camp, the other "students" and I stayed in St. John's Residence, which is next door to the college.

Our days at camp consisted of approximately two hours of learning new math concepts, such as bases, series, sequences, and congruences. Another three and a half hours were spent on trying to answer questions using the concept that we had just learned in the previous class. To add to the fun, we also had a lot of free time in which we could spend our time how we pleased or work on our math. An hour and a half was also spent on sports each day. The sports that we played were soccer, volleyball and ultimate frisbee, and almost everybody participated in them. (If you ask me, the sports were really fun because it had everybody in one big group and we were all enjoying ourselves!) Now let's not forget about the meals. The meals were great!! With breakfast in the residence cafeteria, and lunch and supper at the Daily Bread Cafe, I'm not one to complain.

During two of the four evenings that we were at camp, we were also given presentations from other math professors at the university. The presentations were on polyhedra and fractals . I found both very interesting and many others did too. An off-campus excursion to Fort Whyte also added to everyone's fun for the week. Our other off-campus excursion took us to Dairy Queen. That was a great idea and what made it even better was the fact that it was on our last night at camp.

All right, enough about our days at camp. Time to talk about the people. The people I met at Math Camp were great! I could not have asked to meet nicer people. Our two professors, Dr. Don Trim, the camp co-ordinator and Mr. Bill Korytowski (also known as "Dad" because he was our house-father) were the best! They were a lot of fun to hang around with, they loved to socialize with us, and they were great teachers. (They were also great at sports as we students found out!) The four T.A.'s (math students at the university) were all very nice too. They helped us on our questions when we were "stuck", were our supervisors (and teammates for sports), and did many other things with us. As for the other "students" at camp, they were fantastic! They were easy to get along with, a lot of fun to hang around, and they all had great personalities. There was something unique about everybody.

Looking back at my week at Math Camp now, I think to myself, "Wow! Time really does fly!" It didn't seem right to be going home quite at that time on Friday. Even though the saying goes, "there's no place like home", I felt so at home with everything at Math Camp that I can't wait until I can come back next year.

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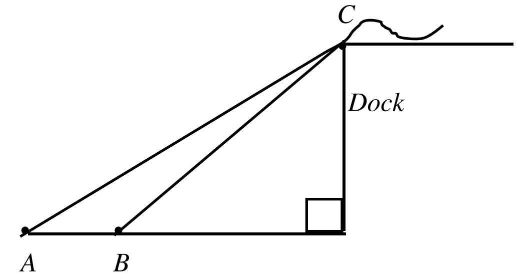
Solution to the boat problem from page 1:



We know that $AC = 3 + BC$. Since the shortest distance between A and C is along the straight line joining them, $AB + BC$ is greater than AC, that is, $AB + BC$ is greater than $3 + BC$. It follows that AB is greater than 3.

Solution to the rational-irrational problem from page 1:

We know that $\sqrt{2}$ is an irrational number.

Consider the real number $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$.

By a law for exponents, $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{(\sqrt{2})(\sqrt{2})} = \left(\sqrt{2}\right)^2 = 2.$

There are two possibilities: either $\sqrt{2}^{\sqrt{2}}$ is rational or it is not rational. Suppose $\sqrt{2}^{\sqrt{2}}$ is rational then it is an example of an irrational power of an irrational number which equals a rational number.

Suppose $\sqrt{2}^{\sqrt{2}}$ is not rational. Then $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ is an example of an irrational power of an irrational number which equals a rational number, namely 2.

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HOW TO PROVE IT:

Proof by example:
the author gives only the case $n = 2$ and suggests that it contains most of the ideas of the general proof.

Proof by intimidation:
"Trivial".

Proof by vigorous hand waving:
works well in a classroom or seminar setting.

Proof by cumbersome notation:
best done with access to at least four alphabets and special symbols.

Proof by exhaustion:
an issue or two of a journal devoted to your proof is useful.

Dana Angluin, Siagact News, Winter-Spring 1983, Vol 15 #1

Annual High School Problem Solving Workshop

Stephanie Olafson
Institute of Industrial Mathematical Sciences

Each year, the Institute of Industrial Mathematical Sciences and the Department of Mathematics at the University of Manitoba jointly organize a workshop for selected students in Senior 2, 3 and 4 from high schools in and around Winnipeg. The main purpose is to provide training for the Cayley, Fermat and Euclid Mathematics Competitions and to assist students in writing the Manitoba Senior 4 Competitions. In addition, the workshops also improve students' problem solving abilities, bring students with a passion for mathematics together to work individually and in groups, and allow the high school students to meet enthusiastic undergraduate students, graduate students and faculty members in the mathematical sciences.

The training is held in Machray Hall at the University of Manitoba, and consists of sessions on four consecutive Saturdays. The sessions run from 9:00 a.m. to 3:00 p.m., with lunch being provided to all participants during a one-hour break. These sessions are devoted to theoretical and practical aspects of problem solving. Problems are selected from various sources including past Cayley, Fermat and Euclid competitions. Class sizes are kept at reasonable levels and each class has more than one instructor to ensure fun is had by all participants.

Principals of high schools within a 50-km radius of the Fort Garry Campus are asked to consult the Heads of their Mathematics Departments in the selection of one or more students for participation in the workshop to the fullest extent. Interested students should also bring this to the attention of their math teachers.

This year's workshop is to be held held January 25, Feb 1, 8, and 15, 2003.

For further information about the workshop, please contact the Institute of Industrial Mathematical Sciences at 474-6724 or iims@UManitoba.CA.

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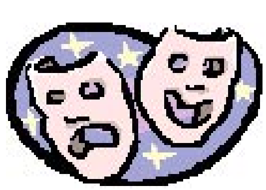


What keeps a square from moving???

Square roots of course!!!!!

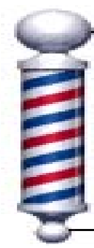
Cool Websites

Mathematical Paradoxes and Fallacies

R. Padmanabhan
Mathematics Dept

A paradox is an assertion that is essentially self-contradictory, though based on a valid deduction from acceptable premises. What this means, more or less, is that there is some logical problem going on; either the deduction isn't really valid, or the premises aren't really acceptable. Alternately, the premises and the deduction are fine, and the universe in which it is operating really is self-contradictory.

In a certain village there is a man, so the paradox runs, who is a barber. A prominently displayed signboard in front of the barber shop says



This barber shaves all and only those men in the village who do not shave themselves.

This seems fair enough, and fairly simple until, a little later, the following question occurs to you - does the barber shave himself? If he does, then he mustn't, because he doesn't shave men who shave themselves, but then he doesn't, so he must, because he shaves every man who doesn't shave himself... and so on. Both possibilities lead to a contradiction.

This is the famous Barber's Paradox**, discovered by mathematician and philosopher Bertrand Russell, at the beginning of the twentieth century. As stated, it seems quite simple, but in fact, restated in terms of so-called "naive" set theory, the Barber's paradox exposed a huge problem, and changed the entire direction of twentieth century mathematics. If you want to learn about the history of such paradoxes, why not visit the websites given below and follow the links to get even further connections to related topics.

http://mathworld.wolfram.com/Paradox.html
http://www.bbc.co.uk/dna/h2g2/A581096
http://mathforum.org/discuss/sci.math/m/190650/190682
http://info-pollution.com/math.htm
http://www.freakytim.co.uk/stuff/paradox.asp

Instead of looking at websites, if you rather prefer learning through reading books, then there is a very nice volume on this topic:

Bryan Bunch: Mathematical Fallacies and Paradoxes,
Van Nostrand Reinhold, Toronto.

Your school library may have it. It contains stimulating and thought-provoking analysis of a number of the most interesting intellectual inconsistencies in mathematics.

**There is a rather easy (and an anticlimax) 21st century solution to the Barber's Paradox, which doesn't require the opening of any nasty can of set-theoretic worms. Just make the barber a woman...□, sorry.

NEXT ISSUE: Paradoxes in Geometry

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Professor Snape's Challenge

Dan Potter
4th Yr Mathematics

As an avid Harry Potter fan, I naturally jumped at the chance to create, for publication, a solution to the challenging problem presented at the end of J.K. Rowling's "Harry Potter and the Philosopher's Stone" [Rowling, 1997]. What follows is a variation of the problem presented to the characters Harry Potter and Hermione Granger by Prof. Severus Snape.

The educational value of the solution presented is that it requires the use of logical reasoning, and shows how a complex problem can be broken down into manageable cases.

Problem :

Harry and Hermione are locked in a room containing a table and two doors. One door leads onwards, the other leads back. Sitting in a row on the table are seven potions in bottles of different sizes. Also on the table is a note explaining that one of the potions, when drunk, will allow them to continue forward through the door, another will let them move back through the other door, two are wine and three are poison.

To help them choose the potion which will allow them to move forward, the note also gives four clues:

1. *on the immediate left of the wine, poison will always be found;*
2. *the first and last potions in the row are not the same, and neither of them is the potion that will allow them to move forward;*
3. *the potions in the largest and smallest bottles are not poison;*
4. *the second potion from the left of the row is the same as the second potion from the right.*

Hermione correctly determined that the smallest bottle contained the potion that will allow them to move forward. What are the possible ways that the potions could have been arranged?

Solution:

Label the seven potions $p1$, $p2$,.., $p7$ where $p1$ is on the left end of the row and the other potions follow in order. Let W denote wine, P denote poison, B denote the potion that will allow Harry and Hermione to move back, and F denote the potion that will allow them forward. Let $\oplus$ denote the largest bottle and let Θ denote the smallest.

To the student reading the solution, I encourage you to follow along each of the first few cases with a pencil and paper, keeping track of which possibilities are eliminated and which remain.

A handy way to do this is by drawing a four-row by seven-column grid. In each of the seven cells in the top row enter the letter 'F'. In the second row, enter the letter 'B' in each cell, 'W' in the cells in the third row and 'P' in the cells in the bottom row.

As you work through a particular case, you will very often determine that a particular one of the seven potions is not F or is not B or is not W or is not P. You may record your discovery by crossing out the appropriate letter in the appropriate column.

For example, the second clue tells us that $p7$ is not F. So, we would cross out the 'F' in the seventh column of the grid. Keeping track of this will help us later on.

Now, there are many different possible orderings of the seven potions, but we may eliminate many of them using the four clues:

> Wine must always have poison to its left. Since $p1$ has nothing to its left, $p1$ cannot be W. $p1$ is also given not to be F. So $p1 \in \{B,P\}$. Since there is only one F and one B, and $p2$ and $p6$ are given to be duplicates, $p2$ and $p6$ cannot be F or B. So $p2 \in \{W,P\}$ and $p6 \in \{W,P\}$.
> Since $p7$ is given not to be F, $p7 \in \{B,W,P\}$.

We can further reduce the problem by considering the locations of the largest and smallest bottles, which we know are not poison. Since Hermione determined that the smallest bottle was F, and we've determined that $p1$, $p2$, $p6$ and $p7$ are not F, we know that the smallest bottle must be one of $p3$, $p4$ and $p5$.

Let case (a,b) denote the case where $\oplus$ (the largest bottle) is in the a^{th} position and Θ(the smallest bottle) is in the b^{th} position. Since we've determined that $p1$, $p2$, $p6$ and $p7$ are not F, we know that what Harry and Hermione saw on the table was a case where $b \in \{3,4,5\}$. Hence, we can eliminate immediately all cases where $b \in \{1,2,6,7\}$. That leaves us to examine the cases :

(1,3), (2,3), (4,3), (5,3), (6,3), (7,3),
(1,4), (2,4), (3,4), (5,4), (6,4), (7,4),
(1,5), (2,5), (3,5), (4,5), (6,5), (7,5).

Case (1,3):

We know that $p1 \in \{BP\}$. Since $p1 = \oplus$, $p1 \neq P$. Therefore, $p1 = B$. Since $p1 = B$, $p2$, $p3$, ...,$p7$ are not B.

According to the first clue, $p1 \neq P$ $p2 \neq W$. Since we knew that $p2 \in \{W,P\}$, $p2 = P$. $p2 = P \Rightarrow p6 = P$, by the fourth clue.

If $p7 = P$ then we'd have $p2 = p6 = p7 = P$. But this would not allow us to have poison to the left of both wines, so $p7 \neq P$, by the first clue. We've shown that $p7 \neq B$, $p7 \neq F$ (clue 2) and $p7 \neq P$. Therefore $p7 = W$. From here, we have no further information about $p3$, $p4$ and $p5$, except that $p3 = \Theta$ and therefore $p3 \neq P$.

We cannot determine a unique solution. Therefore, Case (1,3) must not have been what Harry and Hermione saw on the table. The best that Harry and Hermione could safely do in this situation is to drink $p3$, since we know that $p3$ is not P. If $p3$ turns out to be F, then the puzzle is solved. If $p3$ turns out to be W, a solution cannot be determined.

Case (2,3):

Since $p2 = \oplus$, $p2 \neq P$. Therefore, $p2$ and consequently $p6$, are W. Therefore $p1 = p5 = P$. Since $p1 = P$, $p7 \neq P$ (by clue 2). We know that $p7 \neq F$ (also by clue 2). We know that $p7 \neq W$, since we've already accounted for both wines. So, $p7 = B$. We still need to account for a single P and F. We have not yet determined the contents of $p3$ and $p4$. But since $p3 = \Theta$, $p3 \neq P$. Therefore $p3 = F$.

Case (2,3) yields a solution, hence this might have been the configuration that Harry and Hermione saw on the table. *(continued on page 6)*

Case (4,3):

Since $p3 = \Theta$ and $p4 = \oplus$, $p3 \neq P$ and $p4 \neq P$. By the first clue, $p4$ and $p5$ are not W. We divide this case into two subcases:

Subcase α:
Assume that $p2 = p6 = W$. Then $p1 = p5 = P$ (by clue 1). $p1 = P \Rightarrow$ p4 $\neq$ P. $p7 \neq W$, since both wines are already accounted for. So $p7 = B$. But now we've determined that $p1 = p5 = P$, and $p2$, $p3$, $p4$, $p6$ and $p7$ are not P. But there are three posion bottles, so case α is clearly impossible.

Subcase β:
Assume that $p2 = p6 = W$. In this case, if $p7 = P$, then there are not two poisons available for wine to be on the right of. So $p7 \neq P$. The available positions for the single other poison are $p1$ and $p5$. Whichever position it's in, it will have poison to it's right. Hence, wine must be to the right of $p2$ and $p6$. This situation yields no clear solution. However, it is safe to drink $p4$, since $p4 = \oplus$. If $p4$ turns out to be F, the puzzle is solved. If $p4$ turns out to be B (we've eliminated the other possibilities) then $p1$ must be P. Then $p5$ would be the only undetermined potion. By process of elimination, $p5$ would be F.

Hermione was able to solve the problem without needing to drink any of the potions, so case (4,3) must not have been what she and Harry saw on the table. A simpler method of finding F is just to drink $p3$ and $p4$ right away. If one of them turns out to be F, the puzzle is solved. If not, $p5$ must be F, because we've eliminated F from being a possibility for every other potion.

Case (5,3):

$p5 = \Theta \Rightarrow$ and $p3 = \oplus$ therefore these two potions are not poison. This tells us that $p4$ and $p6$ are not wine. $p6$ is not wine means that $p6$ must be poison, and therefore $p2$ is poison. This leaves us with three possible positions for the remaining poison: $p1$, $p4$ and $p7$. If $p7 = P$ then there are not two available positions to the right of poisons, hence wine will not always have poison to its left. So $p7 \neq$ P. But assuming that the other poison in $p1$ or in $p4$ gives us no information about $p3$ and $p5$. Hence, this case yields no distinct solution.

The best that Harry and Herminone can do is drink potions $p3$, $p5$ and $p7$. This arrangement has no distinct solution.

Case (6,3):

$p6 = \oplus \Rightarrow p6 \neq \mathrm{P} \Rightarrow p6 = W$ and $p2 = W$. Therefore (by clue 1) $p1 = p5 = P$. Since $p1 = P$, $p7 \neq P$ (by clue 2).

We've determined five of the seven potions, leaving $p3$ and $p4$ as yet unknown. However, we still need to account for another poison. Since $p3 = \Theta$, $p3 \neq P$. So $p4 = P$ and $p3 = F$.

Hence, Harry and Hermione may have seen this configuration on the table.

Case (7,3):

$p7$ is not poison. Also, $p3$ are not poison, so $p4$ is not wine. We have no information about $p1$, $p2$, $p4$ or $p5$ and so cannot determine a solution.

As such, this was not the configuration.

Case (1,4):

$p1 \neq P \Rightarrow p1 = B \Rightarrow p7 \neq B$. Now, $p4 \neq P$ and $p4 \neq B$, so $p4 \in \{W,F\}$. We canot solve this case without first drinking a potion. Drink $p4$. If $p4 = W$ then $p3 = P$. Also, since there are only two wines, $p4 = W$ means that $p2$ and $p6$ cannot both be wine. Since $p2$ and $p6$ are the same, they must be poison. The only other position that is to the right of a poison and is still undetermined is $p7$. This leaves only $p5$ undetermined. So $p5$ must be F. However, we did not know to start out with $p4 = W$. We had to first drink $p4$. $p4$ could also have been F.

In either case, we are able to figure out which potion is F, but Hermione didn't need to drink any potions before coming to a conclusion. So this must not have been the configuration that Hermione saw on the table.

Case (2,4):

Since $p2 \neq P$, $p2 = W$. Therefore $p6 = W$. $p2 = p6 = W \Rightarrow p1 = p5 =$ P. $p1 = P \Rightarrow p7 \neq P$. Since we've already accounted for both of our wines, $p7 \neq W$. $p7 \neq F$ (from clue 2). So, $p7 = B$.

A single P and F remain to be determined. There are two positions undetermined: $p3$ and $p4$. But $p4 = \Theta$, therefore it is not the poison. Hence, it must be F.

Since we were able to determine a solution, this might have been the arrangement.

Case (3,4):

$\oplus$ and Θ play identical roles. So, the proof for case (4,3) is sufficient to show that this is not the arrangement that Harry and Hermione saw.

Case (5,4):

$p4$ and $p5$ are not poison, so $p5$ and $p6$ are not wine. $p6$ is not wine, so $p6$ and $p2$ must be poison. If $p7$ were poison, then there would be a wine without a posion to its left, since $p6$ would have poison to its right and $p7$ is at the right end of the row. So $p7$ is not poison.

There are now only two possible positions for the remaining poison: $p1$ and $p3$. If $p3 = P$ then $p4$ and $p7$ are the locations of the wine. All the locations of all poisons have been determined and $p1$ is not one of them. So the remaining possibility for $p1$ is B. This leaves only $p5$ undertermined, so $p5 = F$. If $p1 = P$ then $p3$ and $p7$ are the locations of the wine.

We are then left with no information about $p4$ and $p5$, except that one is F and the other is B. So, in case (5,4), we can determine that F is one of $p4$, $p5$, but nothing more.

Case (6,4):

Since $p6$ is not poison, $p6$ must be wine. Therefore $p2$ must be wine. This tells us that $p1$ and $p5$ are poison. $p1$ is poison, therefore $p7$ is not poison. Both wines are accounted for, and $p7$ is known not to be F. So $p7 = B$. This leaves us with a single poison and F undetermined. The two positions that are not determined are $p3$ and $p4$. $p4 = \Theta$, therefore is not poison. So $p4 = F$.

We've found a solution where F is the smallest bottle, so this arrangement of bottles may have been what Harry and Hermione saw on the table.

Case (7,4):

$p7$ is not poison. Since $p4$ is not poison, $p5$ is not wine. We cannot determine a unique solution from here. However, we know that $p4$ is safe to drink, so drink $p4$. If $p4$ turns out to be F, the puzzle is solved. If $p4$ turns out to be W, then it is not possible that both $p2$ and $p6$ are wine, since there are only two wines in total. So $p2 = p6 = P$. Since $p4 = W$, $p3$ must be the other poison. All poisons are now accounted for. $p1$ must be B, since we determined at the beginning that $p1 \in \{B,P\}$. $p7$ is the only position that is undetermined and that has a poison to it's left. Therefore $p7 = W$. We've now determined every potion except for $p5$. So $p5$ must be F.

If $p4$ turns out to be B, then $p7$ cannot be B. Since $p7$ (clue 2) is not F and not P (since its the lagest bottle), $p7 = W$. $p1 \in \{\mathrm{B,P}\}$ and $p4 = B$, so $p1 = P$. Also, $p7 = W$ means that $p6$ and therefore $p2$ are poison. $p3$ is now the only undetermined position which has a poison to its left, so $p3$ must be the remaining wine. The only undetermined position is now $p5$, so $p5 = F$.

We are able to solve this arrangement, but not without drinking $p4$. Hermione solved her arrangement without drinking any potions, so this must not have been the arrangement.

Case (1,5):

$p5 \neq P$, therefore $p6 \neq W$. But $p6 = \{W,P\}$, so $p6 = P$. Consequently, $p2 = P$. Also, $p1 \in \{B,P\}$ and since $p1 = \oplus$, $p1 \neq P$. i.e. $p1 = B$. $p7 \neq P$, because otherwise there would not be two undetermined positions with poison on their left sides. Since $p1 = B$, $p7 \neq B$. So, $p7 = W$.

We have no further information about $p3$, $p4$ and $p5$, except that $p5$ is not poison. So, we cannot determine a unique solution. However, we know that $p5$ is safe to drink. If we drink $p5$, and it turns out to be F, the puzzle is solved. If $p5$ turns out to be wine, then $p4$ must be poison, and the only remaining position, $p3$ must be F.

Because we had to drink one of the potions to solve this arrangement, this is not the arrangement Harry and Hermione saw on the table.

Case (2,5):

We know that $p2$ is not poison, and $p2 \in \{W,P\}$ so $p2$ must be wine. Therefore $p6$ is also wine. However, $p6 = W$ implies that $p5 = P$. But in this case $p5 = \Theta$ and is therefore not poison.

Hence, this arrangement is in contradiction with the clues.

Case (3,5):

$\oplus$ and Θ play identical roles. So, the proof for case (5,3) is sufficient to show that this is not the arrangement.

Case (4,5):

Once again, $\oplus$ and Θ play identical roles. This is not the arrangement.

Case (6,5):

$p6 \neq P$ and $p6 \in \{W,P\}$, so $p6 = W$. Therefore $p5 = P$. But $p5 = \Theta$, and is therefore not poison.

Hence, this arrangement is in contradiction with the clues.

Case (7,5):

$p6 \neq P$ and $p6 \in \{W,P\}$, so $p6 = W$. Therefore $p2 = W$. Since to the right of wine we always find poison, $p1 = p5 = P$. Also, $p7 \in \{B,W,P\}$, but since there are only two wines, $p7 \neq W$. Since $p7 = \oplus$, $p7 \neq P$. So, $p7 = B$. Undetermined still are $p3$ and $p4$. Since we have no information about either of them, we cannot determine which of them is poison and which is F.

Hence this arrangement yields no distinct solution, and must therefore not have been the arrangement that Harry and Hermoine saw on the table.

Conclusion:

We found four arrangements of the potions that make the puzzle solvable, with the smallest bottle as F (smallest and largest bottles are underlined):

P<u>WF</u>PPPWB

PW<u>F</u>PPW<u>B</u>

P<u>W</u>P<u>F</u>PWB

PWP<u>F</u>P<u>W</u>B

Citation: [Rowling, 1997] Rowling, J.K., Harry Potter and the Philosopher's Stone, Bloomsbury Publishing, London, England, 1997, pp.206-208.

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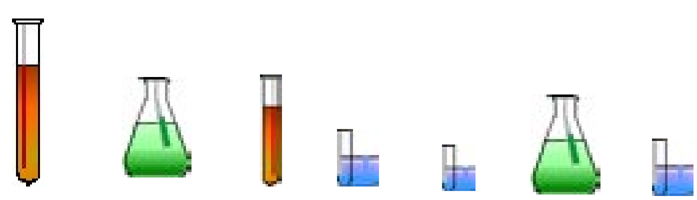

PROBLEM CORNER

D. Trim
Mathematics Dept

Dear Readers:
Welcome once again to PROBLEM CORNER. Here is the problem from the last column and its solution. If we select four different digits from the set {1,2,3,4,5,6,7,8,9} they can be arranged to produce 24 different four-digit numbers. Suppose that these 24 numbers are added to produce a sum. What is the maximum number of distinct primes that will divide every such sum?

Let a, b, c, and d be the four different digits chosen. When all 24 different four digit numbers are formed from these and added, each of a, b, c, and d will appear 6 times in the thousands, hundreds, tens, and units positions. This means that the sum of the 24 numbers will

$$6(a + b + c + d)\ (1000 + 100 + 1) = 6\ (1111)\ (a + b + c + d) = 2 \bullet 3 \bullet 11 \bullet 101\ (a + b + c + d).$$

Thus the four primes 2, 3, 11, and 101 divide the sum for any choice of a, b, c, and d. To find the maximum number of primes that divide all such sums, we should determine choices of a, b, c, and d that yield the minimum number of additional prime divisors. We would ask if there is a choice that yields no additional prime divisors, and the answer is yes. If we chose $a = 1$, $b = 2$, $c = 3$ and $d = 5$, then $a + b + c + d = 11$, then the sum is

$$2 \bullet 3 \bullet 11 \bullet 101 \bullet 11 = 2 \bullet 3 \bullet 11^2 \bullet 101.$$

The number of prime divisors is 4.

Congratulations to **Ryan Holm** from **St. Ignatius High School** in Thunder Bay for submitting a correct solution. It is not the first solution submitted by Ryan. I firmly believe that more of you are solving these problems. Send them to me in care of

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Here is your problem for next time:
Jordan, an electrician, spends a whole day installing a communication cable from the basement of a building to the 13th floor. The cable consists of a bundle of 51 individual wires, somewhat like a larger version of a telephone cord. After "finishing" the job, Jordan has to connect the individual wires to various telephones and computer devices, but notices that the wires are indistinguishable because they are all the same colour. Jordan knows the purpose of each wire at the basement end, but does not know the corresponding wires on the 13th floor. Jordan wonders how to solve this problem with minimum effort. That is, for each of the 51 wires in the basement, Jordan needs to identify the corresponding wire on the 13th floor with a minimum number of trips between the basement and the 13th floor. Jordan only has a continuity tester (that is, it indicates when two wires are connected together making a circuit when connected by the circuit tester). What is the minimum number of trips from one end of the cable to the other than Jordan has to make?

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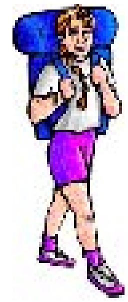

MathCamp 2002....... A Success!!!

D. Trim
Mathematics Dept

The Department of Mathematics conducted its first Mathematics Camp on August 18-23, 2002.

Twenty students from grades 8, 9 and 10, who stayed in residence at St. John's college, were immersed in an intensive program of mathematics and its applications. Two of the students were from Grandview, one from Petersfield, one from Riverton, and sixteen from Winnipeg, six girls and fourteen boys.

On Monday through Thursday, there were two hours of class time, one in the morning and one in the afternoon, after which students applied their understanding to extensive problems sets on the theme of the day. Topics included arithmetic in different bases, sequences and series, finite and infinite, linear congruences, and methods of proof in mathematics. Students were also given an additional set of problems typical of what they might encounter in provincial and national competitions.

Other activities included an excursion to the Fort Whyte Center and team sports such as soccer, volleyball and ultimate frisbee.

At the end of camp students filled out questionnaires evaluating the camp.

The camp was given an average rating of **9.1 out of 10!**

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A somewhat advanced society has figured how to package basic knowledge in pill form. A student, needing some learning, goes to the pharmacy and asks what kind of knowledge pills are available. The pharmacist says *"Here's a pill for English literature."* The student takes the pill and swallows it and has new knowledge about English literature! *"What else do you have?"* asks the student. *"Well, I have pills for art history, biology and world history,"* replies the pharmacist. The student asks for these, and swallows them and has new knowledge about those subjects. Then the student asks, *"Do you have a pill for math?"* The pharmacist says *"Wait just a moment,"* and goes back into the storeroom and brings back a whopper of a pill and plunks it on the counter. *"I have to take that huge pill for math?"* inquires the student. The pharmacist replied, *"Well, you know math always was a little hard to swallow."*

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