MANITOBA MATH LINKS

UNIVERSITY OF MANITOBA

Volume 1, Issue #3, Spring 2001

University of Manitoba Outreach Project

Published by the Department of Mathematics

THE WOMAN AT THE WELL

R. Craigen

A good mathematician knows that every answer leads to a new question that goes one step further - to push a metaphor, the well of good problems is deep and plentiful. But there is an art to finding a good "next question."

A woman comes to the village well, carrying two empty jars. One jar, she knows, will hold exactly 7 *xygs** of water, while the other will hold exactly 3. She requires 5 *xygs* of water for her family's needs that day, and she only has the time to make a single trip. Further, she knows from experience that, while she can begin the return trip with more than 5 *xygs* of water in her jars, she will become very tired on the return trip and will be in jeopardy of dropping one of them — a risk she is unwilling to take.

Therefore, it is essential that she measure exactly 5 *xygs* of water from the well to carry home (distributed, of course, between the two jars). At the well, there are no buckets or other measuring instruments, and she encounters no one who can assist her in making such measurements. Yet she confidently draws water and returns with the desired amount. How?

Of course, you will recognize this classical problem, and some elementary experimentation yields the answer. One solution is to fill the smaller jar twice, each time emptying it into the larger one. Thus, the larger jar contains 6 *xygs* and can admit only one more *xyg*. Fill the smaller jar again and use it to fill the larger one. This leaves 2 *xygs* in the smaller jar. Empty the larger jar and transfer the two *xygs* to it from the smaller jar. Then fill the smaller jar. There are 2 *xygs* in the large jar and 3 in the small one — 5 *xygs* altogether.

Now let us change the story. This time the woman owns the well and is a shrewd businessperson. She realizes that in this dry land the other villagers would pay her to deliver various amounts of *xygs* of water; some will only want 4; others will want 10, and so on. So she sets a reasonable pricing scheme based on the quantity of water ordered, plus cost of delivery. Meticulous in her dealings, she will charge neither too much nor too little, nor promise the sale of an amount she cannot deliver.

The largest quantity she might deliver (at a time) is 10 *xygs*, and the smallest amount — if she can indeed measure it — is 1. So there are 10 possibilities altogether (clearly only integer amounts are possible; while 0 *xygs* is theoretically possible, it makes no sense for this purpose). Of these 10, which can the woman include in her pricing scheme?

A few of the measurable quantities are rather obvious. 3, 7 and 10 correspond to one or two full jars. We have already seen how 5 is attained; 2, 6 and 9 were attained along the way. This leaves 1, 4 and 8. For 8, one might obtain 5 as above, put them all in the large jar, and fill the small jar again. For 1, begin with this 8, fill the large jar from the small one (leaving 1 *xyg* in the small jar) and throw out the contents of the large jar. I leave it to the reader to see how to obtain 4 *xygs* from this point in two steps.

Remarkably, ALL 10 of the possible quantities turn out to be measurable! Perhaps there is something special about the numbers 3 and 7? Would the same thing be true if the jars held 5 and 8 *xygs* respectively, 4 and 14, or some other quantities?

Let us state the general question more clearly: suppose one can measure (positive integer) quantities m and n in two containers, and pass amounts back and forth, either completely filling or emptying a container in each step. What quantities can be measured (and subsequently contained) by these containers?

To be more concrete, let us try this with $m = 5$ and $n = 8$. The possible values are 1,...,13. Each line in the table below is obtained, in an obvious way, in a single step from the preceding line.

***xygs refer to a fictional unit of volume*

(continued on page 3)

MANITOBA MATH LINKS WEBSITE

.......www.umanitoba.ca/faculties/science/mathematics

A NOTE FROM THE EDITORS:

We welcome comments from our readers and value their advice. Do you have any suggestions for improving Math Links? Topics for new articles? Drop us a line....either by e-mail or regular mail....to the attention of our Co-ordinator. We enjoy hearing from you...!

DATES TO MARK ON YOUR CALENDAR

JULY 4 - Deadline for mail applications for Science & University I

The Math Links Newsletter is published by the Mathematics Department Outreach Committee three times a year (Fall, Winter & Spring) .

EDITORS: *Prof. R. Padmanabhan*
Prof. R. Craigen
Prof. R. Gaudet
Prof. A. Gumel

CO-ORDINATOR: *S. Kangas*
(kangass@cc.umanitoba.ca)

OUR WEBSITE: *www.umanitoba.ca/faculties/science/mathematics*

MAIL ADDRESS: *Math Links Newsletter*
Department of Mathematics
University of Manitoba
342 Machray Hall
Winnipeg MB R3T 2N2

FAX #: *(204) 474-7611*
TEL #: *(204) 474-8703*

COOL WEBSITES TO EXPLORE

R. Padmanabhan

(1) You've worked out the answer to a very complex calculation and the number you get is .31830988618379 . You suspect that this might be a decimal approximation for some simple expression made up of constants and arithmetic operators and functions, but you don't recognize the number. So you go and ask the Inverse Symbolic Calculator, a project by Simon Plouffe. Enter your number and hit "Run". The ISC works hard and then tells you that your number is probably the reciprocal of π. Amazing! Here is a sample input-output session:
Your input: .31830988618379. The symbolic calculator's output of .31830988618379 was probably generated by one the following functions or found in one of the given tables. Answers are given from shortest to longest description.

Bases constants, Pi, e, sqrt(2), etc...
.3183098861837906 = 1/Pi
Mixed constants with 5 operations
.3183098861837906 = 1/Pi
Mixed constants with 4 operations
.3183098861837906 = 1/Pi*ln(exp(1))

http://www.lacim.uqam.ca/pi/

(2) Symmetry and the Shape of Space is an interesting symmetry site, with a variety of patterns from art and nature. Some patterns are given on which you can test your classification skill.

http://www.geom.umn.edu/~strauss/symmetry.unit/

(3) Do you know that there are only seventeen possible wallpaper designs? These are the so-called wallpaper groups. In this site, each group is represented by a colourful example. This site is put together by David Joyce.

http://aleph0.clarku.edu/~djoyce/wallpaper/

TABLE OF CONTENTS:

(continued from page 1)

m = 5 jar	n = 8 jar	total contents
0	0	0
5	0	5
0	5	5
5	5	10
2	8	10
2	0	2
0	2	2
5	2	7
0	7	7
5	7	12
4	8	12
4	0	4
0	4	4
5	4	9

I leave it to you to finish the table — see if you can get all 13 quantities. In fact, you may observe a pattern in the process above. The small jar is repeatedly filled and emptied, as far as possible, into the large one, and the large jar dumped whenever it is full. If this process is continued, the total contents will eventually run over all possibilities. Can you see why?

Forget for the moment that we throw out 8 *xygs* from time to time, and concentrate on the accumulated total amount of water drawn. In sequence, this will be 0,5,10,15,20,25, and so on. Now let us consider the result of periodically throwing out 8 *xygs*. Every time the total exceeds 8, this will happen. As a result we obtain all remainders left by these numbers upon division by 8: 0,5,2,7,4 and, in the next step, 5 more. So, if we can show that all remainders 0,1,...,7 are thus attained, it would follow that all totals up to 13 are also attained. I claim that this is so.

After drawing water a times, the cumulative total will be $5a$, and the remainder $5a$-$8b$, where b is the appropriate number. Suppose we obtain the same remainder after a and a' steps; that is, $5a-8b = 5a'-8b'$. Rearranging, we have that $5(a-a')=8(b'-b)$. All these numbers are integers, and 8 and 5 have no common factors. It follows that 8 must divide $a-a'$. That is, some multiple of 8 steps have passed between — which proves my claim above.

What if we began with $m = 4$ and $n = 14$? Clearly only even quantities could be measured. Can all even measures (up to 18) be attained? The analysis above would give that $4(a-a') = 14(b'-b)$, or $2(a-a') = 7(b'-b)$. It would follow that 7 different remainders could be attained (since we could conclude that $7(a-a')$ if the same remainder is attained after a and a' steps).

Can you state a general rule that works for all m and n? Just to be sure you understand, consider this problem: the woman wishes to start another similar business in which she can deliver any quantity from 1 to 50, in the same way, using only two jugs which, for practical reasons, should be approximately the same size. What should be the values for m and n?

Now let us go one step further. Suppose the woman has 3 jars, say of sizes 6, 10 and 15. Can every quantity up to 31 be measured? If so, can you devise a simple routine (like the one above) that will eventually attain all possible quantities?

What is the general result for 3 jars of any specified volumes? Four jars? Any number of jars? Surprisingly, each of these is easy to answer, once the two jar problem is fully understood. But, here is one final (more challenging!) variation for you to think about.

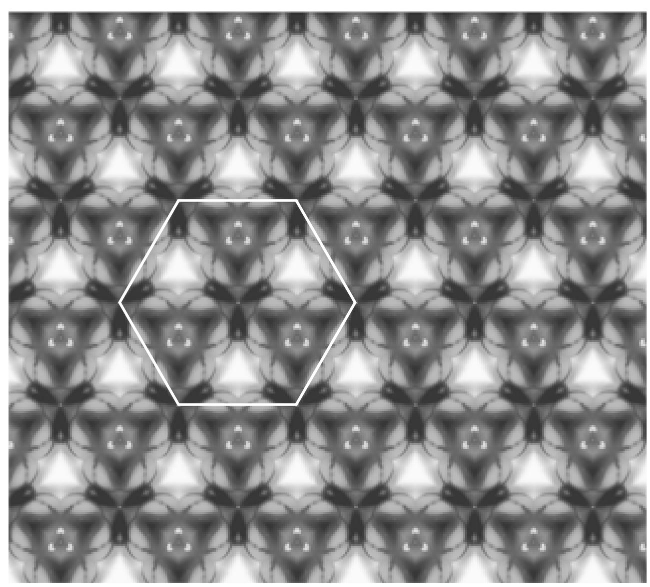
This time the woman decides to use jars whose sizes may be any positive real numbers. She offers to fill orders for any positive real quantity of water (up to the sum of the volumes of the two jars). Of course it is impossible for her to measure ANY real quantity exactly, so instead she claims that the customer may specify to what degree of accuracy the quantities are to be measured (greater accuracy is, of course, more expensive!). Is it possible for her to offer this service with only a finite number of jars? If so, what is the smallest number of jars possible? Once you have determined this, find a method that is guaranteed to eventually fill any order.

Now my jars are full .... but can you think of a good "next question"?

****

TILING OF THE EUCLIDEAN PLANE

R. Padmanabhan



A regular polygon has 3 or 4 or 5 or more sides and angles, all equal. A regular tiling means a tiling made up of congruent regular polygons. (Remember: *regular* means that the sides of the polygon are all the same length. *Congruent* means that the polygons that you put together are all the same size and shape.)

Only three regular polygons tile the Euclidean plane: triangles, squares or hexagons. The above is an example of a regular tiling of the plane with regular hexagons (or with equilateral triangles).

****

PROVING IMPOSSIBILITY

Michael Doob

When we study mathematics, we often have assertions that are proven true. For example, we can prove that every equilateral triangle is also equiangular. A much less common situation is to prove that you *can't* do something. In this article we'll look at an example of such a proof.

In the spirit of the upcoming summer, consider the following problem: at a certain lake there are three cabins, each of which is equipped with a biffy. Since no one likes to wait for the biffy to be vacated, the owners decide to cooperate and have a path from each cabin to each biffy. But since you might go to the biffy at night, and you don't want your neighbours to see you in your night clothes, it is decided that the paths shouldn't cross each other. So the problem is how to lay out the paths from each cabin to each biffy so that they are nonintersecting.

You might think that the particular positioning of the cabins and biffies would play an important role, but that turns out not to be the case. What we'll show is that no matter what the particular positions are, there is no way to constuct the nonintersecting paths.

We'll consider the problem in the following way: make a map by putting dots on a sheet of paper labeled C_1, C_2 and C_3 at the position of the cabins. Do the same with labels B_1, B_2 and B_3 at the position of the biffies, and draw a line where the paths go. One possibility is the following:

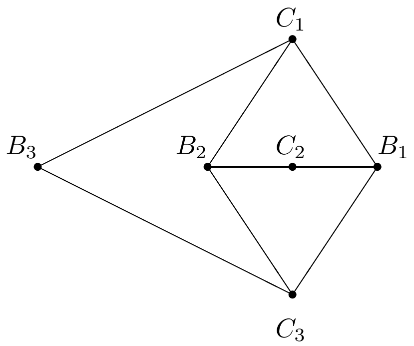


Figure 1: Three cabins and three biffies

Notice that we have almost solved the problem. There is a path (in fact a straight path!) between every cabin and every biffy except for the one missing between C_2 and B_3. But since C_2 is inside the polygon $B_1C_1B_2C_3$ and B_3 is outside of it, although we're only one path shy, we're actually stuck.

Figure 1 is an example of a *graph*. In general, a graph consists of a set of dots, or *vertices* (the singular is *vertex*), and a set of *edges* joining pairs of vertices. When we can draw the graph in the plane so that no two edges intersect (except at vertices), we call the graph *planar*. So Figure 1 is an example of a planar graph.

The graph in Figure 2, traditionally called $K_{3,3}$, has all cabins joined to all biffies, although not in a planar way. What we shall show is that no matter where you put the six vertices and no matter how you join them up, there will always be two edges that intersect (not at a vertex).

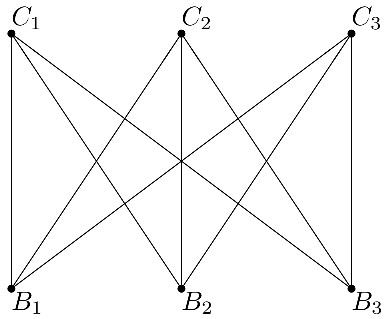


Figure 2: The cabin-biffy graph $K_{3,3}$

To do this we need one more concept. Suppose a graph is drawn in the plane with no two edges intersecting (except at vertices). If the edges and vertices are then deleted from the plane, the pieces of area left are then called the *faces*. The area outside the graph is traditionally called the *face at infinity*, and we denote it by F_∞. The graph in Figure 1 has four faces as shown in Figure 3. We often think of a face as being surrounded by a polygon; the edges and vertices of that polygon are said to be *incident* to the face. In the particular graph in Figure 3, each face (including the face at infinity) is incident to exactly four edges and four vertices. Notice also that each edge is incident to exactly two faces.

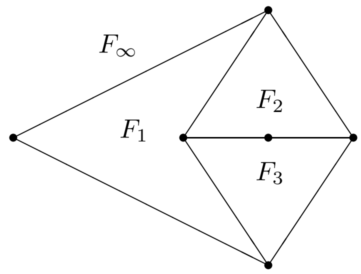


Figure 3: Faces of a graph

For a planar graph G, let v be the number of vertices, e be the number of edges and f be the number of faces (including the face at infinity). We want to focus on the quantity $\chi(G) = f - e + v$. If G is the graph in Figure 3, $v = 6$, $e = 8$, $f = 4$ and $\chi(G) = 2$.

Now suppose we have a planar graph, and suppose we have two vertices incident to a given face with no edge joining them. If we add an edge between the vertices, we split the face in two, so the number of faces increases by one; on the other hand, the number of edges also increases by one and the number of vertices is unchanged, and so, after adding this edge, the quantity $f - e + v$ doesn't change. If we continue adding edges, eventually all faces are triangles. This process is called the *triangulation* of a planar graph.

Next we start deleting vertices and edges from the outside in (that is, incident to the face at infinity). There are two cases:

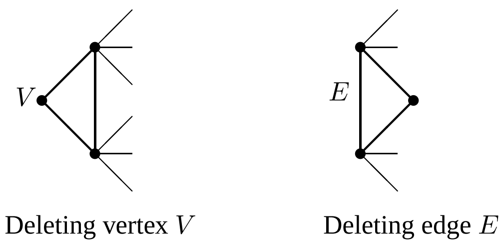


Figure 4: Deletions from the face at infinity

If we have a vertex V incident to the face at infinity that has exactly two edges coming out of it (the left graph in Figure 4) then we delete the vertex and these two edges. This causes the number of faces to decrease by one, the number of edges to decrease by two and the number of vertices to decrease by one. Thus there is no change to the value of $f - e + v$.

If there is no such V, then take an edge E incident to the face at infinity (the right graph in Figure 4). Deleting that edge causes two faces to collapse to one. So f is reduced by one, e is reduced by one and v is unchanged. Hence the value of $f - e + v$ is unchanged. Notice that all faces except for the face at infinity are still triangles.

If we keep deleting vertices and edges in this manner, the resulting graphs get smaller (one triangle at a time), and the value of $f - e + v$ remains unchanged. Eventually the graph becomes a single triangle with $f = 2$, $e = 3$ and $v = 3$. Hence $f - e + v = 2$ for all the graphs. This gives the following theorem:

Theorem 1 *If G is a planar graph with f faces, e edges and v vertices, then*

$$\chi(G) = f - e + v = 2.$$

Next we wish to do some counting: we define $\mathrm{EF}(G)$ to be the number of edge-face incidences. By this we mean the number of pairs (E, F) where E is an edge, F is a face and E is incident to F. For the graph in Figure 3, each edge is incident to exactly two faces. Since there are eight edges, this means the $\mathrm{EF}(G) = 16$. Alternatively, we can note that each face is incident to exactly four edges; since there are four faces, we get $\mathrm{EF}(G) = 16$ once again.

We can make an observation about triangulated planar graphs. Since every face is a triangle, each face is incident to exactly three edges, and hence $\mathrm{EF}(G) = 3f$. Also, each edge is incident to two faces so that $\mathrm{EF}(G) = 2e$. Thus $2e = 3f$ and $f - e + v = 2$, and so $e = 3(v - 2)$. Since any planar graph can be triangulated by adding edges, we conclude the following:

Theorem 2 *If G is a planar graph with e edges and v vertices, then*

$$e \leq 3(v - 2).$$

Now suppose that the graph in Figure 2 could be drawn in the plane. Then from $v = 6$, $e = 9$ and $f - e + v = 2$ we deduce that $f = 5$.

Notice that the graph would have no triangles, since a triangle would include either two of the C_i vertices or two of the B_i vertices, and no two C_i or two B_i vertices are joined by an edge. So each face requires at least one new edge to triangulate the graph. Since there are 9 edges in Figure 2 and 5 faces, there will be at least 14 edges but only 6 vertices in the graph after it is triangulated. This contradicts the result in Theorem 2. So it is impossible to draw the graph $K_{3,3}$ in the plane (no matter how clever we are). Hence we have *proved* that this is impossible.

There is another graph that can't be drawn in the plane. Take five vertices and join every pair by an edge. This graph is called K_5. If it were drawn in the plane, then, since $e = 10$, $v = 5$ Theorem 2 would imply $10 = e \leq 3(v - 2) = 9$, which is not possible.

We have seen that neither K_5 nor $K_{3,3}$ is planar. This obviously implies that any graph containing K_5 or $K_{3,3}$ is not planar either. A famous theorem about graphs proves that, essentially, any graph is not planar must contain within it either K_5 or $K_{3,3}$.

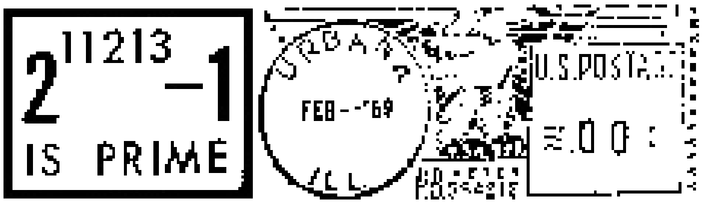
There are other famous problems in which proving impossibility plays a role. The ancient Greeks, in studying plane geometry, wanted to start with a given angle and construct and new angle one third the original size (that is, trisect an angle) using only a compass and straight edge. It was proven (over 2000 years later!) that it is impossible to construct an angle of $20°$, so it's not even possible to trisect a $60°$ angle, much less an arbitrary one. Since we *can* construct a $60°$ angle with a compass and straight edge (as an angle in an equilateral triangle, for example), trisecting the angle is impossible.

Another problem is called "squaring the circle". Given a circle, construct a square with the same area using only a compass and straight edge. This can be proven impossible, too. Some people don't understand the difference between "it is impossible to construct" and "no one knows how to construct", and they (fruitlessly) keep trying to come up with more and more complicated proofs of the unprovable. Sadly, some of these people seem to think that they are victims of some conspiracy of mathematicians. The term "circle-squarers" is sometimes used to describe such people.

There are lots of mathematical problems that we can't solve because we don't know how. A famous example (called the Goldbach Conjecture) asks: is it true that every even number greater than two is the sum of two primes? Most mathematicains would guess yes, but no one can say for sure. So far no one has been clever enough to find a proof.

The moral of this tale is that we need to distinguish between problems that we can't solve because we don't know how to, and problems that can be proven impossible to solve.

Mersenne Primes



Recall that an integer greater than one is called a prime number if its only divisors are one and itself. A Mersenne prime is a prime number of the form Mp = 2^p-1. There are only 38 known Mersenne primes.

The Mersenne number Mp is prime for exponent p = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593.

After the 23rd Mersenne prime was found at the University of Illinois, the mathematics department was so proud that they had their postage meter changed to stamp "2^{11213}-1 is prime" on each envelope.

You can participate in the fascinating project of finding new Mersenne primes at the Great Internet Mersenne Prime Search (GIMPS). Visit http://www.mersenne.org/

R. Padmanabhan

2001 PROBLEM SOLVING WORKSHOP

Kristen Lauren Elkow
Snr 4 - J.H. Bruns Collegiate

For the second consecutive year, I was given the opportunity to attend the math problem-solving workshop presented by the University of Manitoba for high school students. Approximately 150 students, from grades ten to twelve were granted the chance to have their mathematical horizons expanded by peers and teachers from across Winnipeg.

Throughout the workshop I was amazed to see how many high school students were willing to give up four Saturdays to solve math problems! Although I had agreed to come to the math workshop, I had no idea so many people sought mathematical knowledge so readily. I believe what really attracted students to this workshop was the opportunity to learn math concepts beyond school curriculum. Many of the students at this workshop had an extremely high mathematical intelligence, meaning that normal high school math would be unchallenging.

During this four day workshop (which ran for four consecutive Saturdays) various professors gave mathematical diversions. Each of these professors presented us with thought provoking math concepts as complex as prime number theorems, and as simple as the importance of having a “well stocked mind” (Mr. John Barsby, St. John’s Ravenscourt).

Along with lectures, we were given the chance to meet with students from across the city. Individually, I met students from all areas of the city; students from Sisler, St. John’s Ravenscourt and many other schools. Particularly, I enjoyed hearing about the future plans each student has. One girl I met from Maple Grove told me she wanted to become a doctor. Another boy I met wanted to become a bio-systems engineer. What bonded all of the people I met together was the root of their ambitions. Although some students were more gifted mathematicians than others, each student aspired to use their talents to the full.

On a more personal level, I value the opportunity I have been given by J.H. Bruns Collegiate to attend this problem-solving workshop because I enjoy testing my mathematical abilities on a superior scale. Although, being surrounded by future doctors of Mathematics or future Engineering Professors can be very humbling, I have gained satisfaction from the concepts I have learnt. I know through this mathematics workshop I have learnt valuable tools for the coming math contests. I would recommend any high school student (who needs a mathematical challenge) to attend this workshop simply because it gives you perspective on your personal intelligence and capability. There are many clever high school students in the city, which we may not realize in our smaller schools. But, what sets each of these clever people apart from any other gifted student is their willingness to learn more. This is a goal I hope I may be able to achieve in the future: the dedication to the furtherance of my mind.

****

TRANSITION FROM HIGH SCHOOL TO UNIVERSITY CALCULUS I

W. Korytowski

Having been fortunate enough to teach calculus at both the high school and university level, I feel competent to make some comments that future university students may find helpful.

In my experience, students in introductory calculus courses at The University of Manitoba fall into three very distinct categories. The majority of the students are very well prepared for this course. They have a good background in algebra and trigonometry, adapt to the lecture system quickly, attend lectures regularly, and do as many problems as they can. They tend to score very well on the quizzes and examinations.

The second group consists of students who would like to do well but lack the fortitude to attend all lectures, write all quizzes or do enough questions to master the required concepts and techniques. Their presentation tends to be disorganized and lacking in mathematical correctness. They make silly arithmetic errors and rarely study the proofs of the required theorems.

The last group consists of students who lack basic mathematical concepts. The obvious weaknesses are: simple factoring, distinguishing between division by zero and division into zero, operations with fractions, simple trigonometric and logarithmic values, and general functional notation and especially distinguishing between a product and a composition of functions.

Through hard work, any of the students in the above three categories should be able to be successful in the course. It depends on the desire and commitment of each individual student.

The following observations are some additional suggestions which students can take to meet the desired objective of mastering university calculus:

- attend all lectures. The amount of material covered in every lecture is significantly greater than that covered in a typical high school lesson.
- do your questions daily, but review all the work covered to date every weekend. Pretend you are writing an exam every Monday. In the first few weeks this may seem tedious, but if you persevere, this method of over-learning will enable you to review all the previous work in about an hour. The results on your quizzes and examinations will astound you!
- pay more attention to your presentation. The correct answer is usually only a small fraction of the total value of the full solution. If your high school teachers have been deducting marks for poor presentation, thank them, and be grateful that they are concerned for your future.
- study the proofs of all required theorems. It is childish to give away such easy marks! If your high school teachers have been asking you to prove theorems on your tests and examinations thank them again; they have your interests at heart.
- do as many questions from the textbook and previous examinations as you can. Practice is not an option, it is mandatory!
- if your school offers an optional high school calculus course take it if you can fit it in your timetable. The extra time the high school class can spend on topics such as related rate and optimization problems is invaluable when these topics are covered rather quickly at the university level.

Follow the above suggestions in addition to the suggestions in the course handouts that you will receive in your university calculus course, and you will master the course.

****

THRU THE EYES OF A STUDENT

N. Harland
U of M Student

Run! Hide! But they're gonna get you; they won't go away. They will always be there ...waiting .. watching you... they are everywhere ... waiting to strike! Am I being a little over dramatic? Am I???? I hear it everyday .. "I hate math...I can't do it!."

I am the subject everybody loves to hate. I am numbers. I am math. I make the easy difficult. I make the fun horrible. I make the pain....worse.

Is this you? Do you dream about the horrors of math? Well, not dreams.....nightmares. I have the solution. More of it! That's right. The most important question that you will need answered as you prepare for your educational journey is "do I need to take math in university?" Now for all the high school students out there, here is the answer to that most important thought provoking question. Yet, it might be the scariest phrase you will ever hear.

Yes, you need to take math in university! No, it won't end. Why won't it end? WHY???? The answer is simple. It is the answer to the many questions that you will ask yourself over the course of your life. "IT MAKES SENSE"

Now, as difficult as this may be for most people to believe, mathematics is not scary. It is not the end of the world because you have to take math in university. I've met students, who have...well, how do I put this...stunk in math in high school, yet they have thrived in university math.

By the way, if you are thinking this is another attempt by the world of academia to lure unsuspecting students, I am a student. A second year student who sees other students scared of math. Students who are scared to try this course of education because of bad experiences in math, bad marks in math or (I really hate to say this) even bad teachers in math (sadly enough, sometimes it does happen).

You might ask why I care...sometimes I don't even know. Maybe because I see good mathematics students running away from math because of the myth that math is a subject, not a career. Well, I plan to make it a career and I encourage any student to give it a try as well. As I said (or wrote) earlier, you do not have to have a 99.999995% average in math to take this course or that course. You don't have to worry about that scary math professor. You could even have yawned your way through math hoping that one day, in some capacity , you could actually find out that math has a purpose.

I encourage anybody who has ever asked what is the point of taking math to come and ask a professor what can they do with math. I encourage anybody who has ever wondered why they have problems understanding a concept (or many concepts) in math, to walk into your teacher's homeroom or classroom, to ask them what they can do to help. They aren't scary (at least not all of them).I encourage anybody who has been told that you can't make math your career to look into the eyes of a professor teaching a subject they love to a student who tries. Then I dare you to say to yourself "math can't be made into a career". I challenge anybody who said his or her teacher wasn't good enough, wasn't smart enough, didn't know enough, to prove it. Prove it by showing that you can do better.

Finally, I strongly encourage someone who enjoys math, anybody who has thought about extending his or her education in mathematics, to try it. You may regret it sometimes (usually around midterms and final exams) but you won't for long. For what it's worth, I'm glad I have.

I'm Nick Harland...this is what I think.

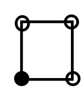
****

PROOF WITH PICTURES

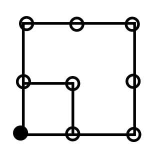
R. Padmanabhan

The square of every natural number is the sum of successive odd numbers. This is the way the whole series of squares can be built, layer by layer, from a single unit. Study the figures given below and construct similar pictures for other squares. Then give a pictorial proof representing n^2 as a sum of odd numbers.

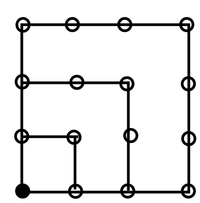
$= 1 = 1 \times 1 = 1^2$



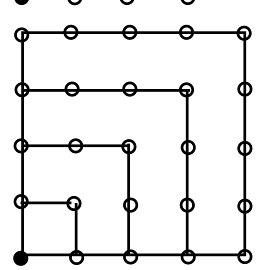
$= 1 + 3 = 2 \times 2 = 2^2$



$= 1 + 3 + 5 = 3 \times 3 = 3^2$



$= 1 + 3 + 5 + 7 = 4 \times 4 = 4^2$



$= 1 + 3 + 5 + 7 + 9 = 5 \times 5 = 5^2$

****

PROBLEM CORNER

D. Trim

Dear Readers:

Welcome back to PROBLEM CORNER. Here is the last problem again and its solution. You are to find all values of the constant c for which the range of the function:

$$f(x) = \frac{x^2 + 2x + c}{x^2 + 4x + 3c}$$

consists of all reals.

If we set $y = \left(x^2 + 2x + c\right)/\left(x^2 + 4x + 3c\right)$, then we can paraphrase the problem as follows: Find all values of c so that for any given y $(-\infty < y < \infty)$, there is at least one value of x satisfying the equation $y = \left(x^2 + 2x + c\right)/\left(x^2 + 4x + 3c\right)$. This suggests that we solve the equation for x in terms of y. First we cross multiply, $\left(x^2 + 4x + 3c\right)y = x^2 + 2x + c$, and then rearrange the equation as $(y-1)x^2 + (4y-2)x + c(3y-1) = 0$.

If $y = 1$, this equation reduces to $2x + 2c = 0$, which has solution $x = -c$, for any c whatsoever. To check that $x = -c$ gives $y = 1$ in the original equation, we set $x = -c$ in the right side, obtaining:

$$\frac{(-c)^2 + 2(-c) + c}{(-c)^2 + 4(-c) + 3c} = \frac{c^2 - c}{c^2 - c} = \frac{c(c-1)}{c(c-1)}.$$

This gives 1 provided c is not equal to 0 or 1, so that these values of c are not acceptable.

When $y \neq 1$, we have a quadratic equation in x. For it to always have at least one solution for x, we require the discriminant to be nonnegative:

$$(4y-2)^2 - 4c(y-1)(3y-1) \geq 0.$$

We must now find values of c which guarantee that this inequality is valid for all y. We rearrange the inequality into the form:

$$(16 - 12c)y^2 + (16c - 16)y + (4 - 4c) \geq 0$$

If we divide by 4:

$$(4 - 3c)y^2 + 4(c-1)y + (1-c) \geq 0.$$

If $c = 4/3$, this inequality reduces to $(4y-1)/3 \geq 0$. This not true for all y. Hence, $c \neq 4/3$.

When $c \neq 4/3$, we have a quadratic in y. For it to always be nonnegative, we require the discriminant to be nonpositive; that is :

$$16(c-1)^2 - 4(4-3c)(1-c) \leq 0.$$

When this inequality is rearranged, we obtain $4c(c-1) \leq 0$. This is true for $0 \leq c \leq 1$. Since $c = 0$ and $c = 1$ have already been eliminated, the possible values of c are $0 < c < 1$. Notice that the solution used properties of quadratic equations, nothing more, but it did require you to think about each quadratic in the appropriate way.

Here is your problem for next time:
Inscribed in a circle is a square. A circle is then inscribed inside the square, a square in the circle, and so on and so on. What is the ratio of the sum of the areas of all the circles to the sum of the areas of all the squares?

Let me encourage you to send a solution to:

S. Kangas
Department of Mathematics
The University of Manitoba
342 Machray Hall
Winnipeg MB R3T 2N2

or by e-mailing :

kangass@cc.umanitoba.ca

I will look at all submissions and print the names and schools of persons who solve the problem correctly and present it in a reasonable way.

****

THE NUMBERS GAME

Paulette Bourgeois

Math has something to do with calculations, formulas, theories and right angles. And every thing to do with real life. Mathematicians not only have the language of the future (they didn't send *Taming of the Shrew* into space, just binary blips) but they can use it to predict when Andromeda will perform a cosmic dance with the Milky Way. It's mathematicians who are designing the intelligent car that knows when you're falling asleep at the wheel or brakes to avoid an accident. It can predict social chaos and the probability of feeding billions. It even explains the stock market and oil prices.

"Excerpt from The Globe and Mail, Thursday, July 13, 2000"