MANITOBA MATH LINKS

UNIVERSITY OF MANITOBA

Volume II, Issue #3, Spring 2002
University of Manitoba Outreach Project
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University of Manitoba 2002 Summer Camp

B. Korytowski
Mathematics Dept

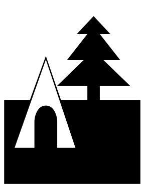
The first (and we expect annual) University of Manitoba Math Camp will be held from August 18 to August 23, 2002.

The Canadian Mathematical Society began actively promoting math camps about five years ago. In 2001, eleven camps were held at various universities from coast to coast. The camps vary considerably in size and length, but they have one thing in common: Mathematics!

With support from the CMS and The University of Manitoba, we hope to enable mathematically talented students to pursue knowledge, in a subject they enjoy, in an environment that encourages and fosters such pursuits. Since all students will be staying in St. John's College Residence, they will have an opportunity to meet and make friends with future mathematicians.

All math camp participants will be students who have completed grade nine and/or grade ten mathematics by June 2002. Information packages will be arriving soon at your school. If you have any questions please contact me by calling 895-3868 or by writing to me at the Manitoba Math Links address.

The camp coordinators this summer will be Prof. Don Trim and myself, from the Department of Mathematics.



We are looking forward to hosting an event which will be stimulating to many eager mathematics students and hope that students from your school will enjoy and benefit from participating in this camp.

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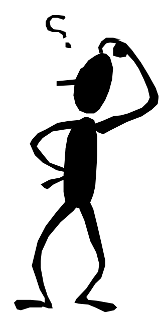
Helpful Hints for 1st Year Students

H. Aldwyn
Mathematics Dept

Students need to keep their eyes open. Signs and posters are everywhere to help students find their way around the mathematics department.

Bulletin boards are founts of information. Students should use them to find out :



- ✓ where their classes are located;
- ✓ who their instructors are;
- ✓ where instructors are located;
- ✓ seminars and math club information;
- ✓ information about new classes;
- ✓ examination information (where and when).

If more help is needed the mathematics office is the next place to go. The Administrative Assistant provides information, advice and sometimes gets to say no.

Students **cannot** get into sections that are full and **cannot** get into courses without the proper prerequisites - 60% or better in Pre-Calculus Math, or 70% in Applied Math 40S for 136.130 or 136.131.

If students are having trouble with an instructor, they should talk to the Administrative Assistant.

Not sure which variety of course you should take? Our department has advisors that you can talk with.

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MANITOBA MATH LINKS WEBSITE

.......www.umanitoba.ca/faculties/science/mathematics

A NOTE FROM THE EDITORS:

We welcome comments from our readers and value their advice. Do you have any suggestions for improving Math Links? Topics for new articles? Drop us a line....either by e-mail or regular mail....to the attention of our Co-ordinator. We enjoy hearing from you...!

DATES TO MARK ON YOUR CALENDAR:

Summer Math Camp Aug 18-23, 2002
Contest Problem Deadline May 24, 2002

The Math Links Newsletter is published by the Mathematics Department Outreach Committee three times a year (Fall, Winter & Spring) .

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SPEAKERS AVAILABLE

If you are interested in having a faculty member come to your school and speak to students, please contact our newsletter co-ordinator..

TABLE OF CONTENTS:

At the end of his course on mathematical methods in optimization, the professor sternly looked at his students and said, "There is one final piece of advice I'm going to give you now - whatever you have learned in my course, never, ever try to apply it to your personal lives!"
"Why?" the students asked.
"Well, some years ago, I observed my wife preparing breakfast, and I noticed that she wasted a lot of time walking back and forth in the kitchen. So, I went to work, optimized the whole procedure, and told my wife about it."
"And what happened?"
"Before I applied my expert knowledge, my wife needed about half an hour to prepare breakfast for the two of us. And now, it takes *me* less than fifteen minutes......"

Cool Websites to Explore

R. Padmanabhan
Mathematics Dept

Magic Squares

A magic square is an arrangement of the numbers from 1 to n^2 (n-squared) in the form of a square matrix, with each number occurring exactly once, and such that the sum of the entries of any row, any column, or any main diagonal is the same. The two well-known magic squares of sizes 3 and 4 are given below:

8	1	6
3	5	7
4	9	2

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

with the magic sums being, respectively, 15 and 34. It is not hard to show that the sum of an n-by-n magic square must be

$$\frac{n\left(n^2+1\right)}{2}.$$

The above 4x4 Dürer magic square has many interesting special properties that are not shared by magic squares in general:

1. The four numbers in the center add to 34.
2. The four squares in the corners add to 34.
3. The four squares, in each of the four corners, add to 34.

The Internet is full of sites dealing exclusively with magic squares and their arithmetical, geometric and other esthetic properties. Just go to 'Google' or your favourite search engine and ask for "magic squares" . You will get about 200+ sites where this topic is discussed in varying degrees of sophistication. Some of them give formulae for constructing pseudo magic squares with a given sum. Say for example, you want to send a greeting card to your teacher who is reaching 38, you may simply add 1 to each number in the above 4-by-4 magic square and make your own mathematical card for your teacher. If you play with magic squares then you will be convinced that math is not just for "Squares".

Some sites you might like to visit are:

http://web.idirect.com/~recmath/ms04.html
http://www.kanadas.com/puzzles/magic-square.html
http://user.chollian.net/~brainstm/MagicSquare.html
http://www-math.mit.edu/~hderksen/magic.html

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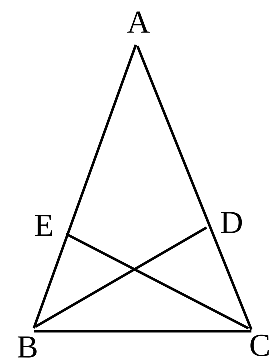
The Lasting Legacy of Ludoph Lehmus

Diane and Roy Dowling
Mathematics Dept

In the early nineteenth century an interesting problem came to the attention of those who enjoyed geometry. It has been said of this problem that its beauty lies in the simplicity of its statement and in the difficulty of its solution. Before looking at it let's consider a problem you may be familiar with:



If in the accompanying diagram AB = AC, BD bisects $\angle ABC$ and CE bisects $\angle ACB$ prove that BD = CE.

When you have solved this problem you have proved the statement:

If a triangle is isosceles then two of its internal bisectors are equal.

About 1840 a question occurred to a Berlin professor, Ludolph Lehmus : is the converse of this statement true?

The converse is : *If two internal bisectors of a triangle are equal then the triangle is isosceles.*

People have thought about the properties of triangles for thousands of years so it is amazing that there is no record of anyone considering this converse before Professor Lehmus did. He approached Jacob Steiner with his question. This famous geometer was soon able to establish the truth of the converse above and it came to be known as the Steiner-Lehmus Theorem. Before very long Professor Lehmus himself found a nicer proof. Since that time geometry hobbyists have been fascinated by the search for simple and neat proofs of the theorem. You can see some of these proofs on the website:

http://www.mathematik.uni-bielefeld.de/~sillke/PUZZLES/steiner-lehmus

In the 1960s Martin Gardner, magician and puzzle enthusiast, who regularly contributed to the *Scientific American*, discussed the Steiner-Lehmus Theorem in one of his columns. This column stimulated a lot of interest and hundreds of readers sent in their own proofs. Martin Gardner examined all these proofs and selected his favourite. This very nice proof was presented by two British engineers, G. Gilbert and D. MacDonnell. A few years later someone searched for the proof that Ludolph Lehmus had found over a hundred years previously and discovered that it was essentially the same as that of Gilbert and MacDonnell! If you would like to see their proof go to the website:

http://poncelet.math.nthu.edu.tw/disk5/js/geometry/geometry.html

The publication of the Steiner-Lehmus Theorem not only got people trying to find neat proofs of the theorem itself but also got them thinking about variations on its theme. For example, is there a corresponding theorem about internal trisectors?

(continued on page 4)

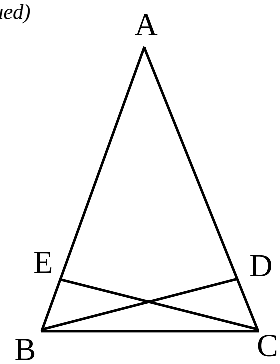
(*The Lasting Legacy of Ludoph Lehmus continued)*

That is, if in this diagram

$\angle CBD = \frac{1}{3}\angle CBA$,

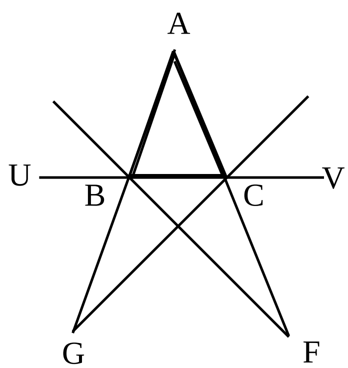
$\angle BCE = \frac{1}{3}\angle BCA$ and $BD = CE$,

can it be shown that $AB = AC$? The answer is yes. In fact, the $\frac{1}{3}$ may be replaced by any fraction between 0 and 1. The proof is not easy.



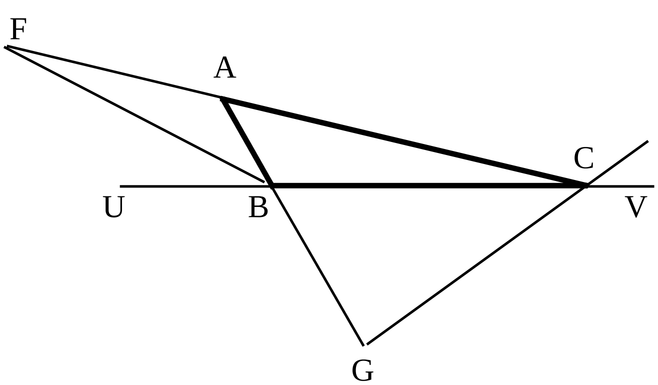
Another variation involves exterior angles of a triangle. For a triangle ABC, if the bisector of an exterior angle at B meets the side AC extended at the point F then the line segment BF is called the external bisector at B. For the triangle ABC in the next diagram the bisector of the exterior angle $\angle ABU$ meets the side AC extended at F so BF is the external bisector at B. The bisector of the exterior angle $\angle ACV$ meets the side AB extended at G so CG is the external bisector at C.

It is not hard to prove that if AB = AC then BF = CG. In other words, if a triangle is isosceles then two of its external bisectors are equal. The converse of this statement is :



If two external bisectors of a triangle are equal then the triangle is isosceles.

At first sight this statement looks very plausible. However, it is not true. A. Emmerich pointed out the surprising fact that a triangle whose interior angles are $132°$, $36°$ and $12°$ has two of its external bisectors equal. A triangle having these angles is referred to as an Emmerich triangle. To see why an Emmerich triangle has two equal external bisectors consider the following diagram .



(Note: diagram not drawn to scale)

Triangle ABC is an Emmerich triangle with $\angle CAB = 36^o$, $\angle ABC = 132^o$ and $\angle BCA = 12^o$. The bisector of the exterior angle $\angle ABU$ meets the side AC extended at F so BF is the external bisector at B. The bisector of the exterior angle $\angle ACV$ meets the side AB extended at G so CG is the external bisector at C. We will show that the external bisector BF equals the external bisector CG .

$$\angle FBA = \frac{1}{2}(180^o - 132^o) = 24^o;$$

$$\angle FBC = \angle FBA + \angle CBA = 24^o + 132^o = 156^o.$$

Now consider triangle BCF:

$\angle BCF = 12^o$ and

$\angle BFC = 180^o - \angle FBC - \angle BCF = 180^o - 156^o - 12^o = 12^o$.

Since $\angle BFC = \angle BCF$, triangle FBC is isoceles with BF = BC.

Now consider triangle BCG:

$$\angle BCG = \frac{1}{2}(180^o - 12^o) = 84^o, \angle GBC = 180^o - 132^o = 48^o.$$

$$\angle BGC = 180^o - \angle BCG - \angle GBC = 180^o - 84^o - 48^o = 48^o.$$

Since $\angle GBC = \angle BGC$, triangle BCG is isoceles with CG = BC. Since BF and CG both equal BC these two external bisectors are equal to each other.

Over 160 years have passed since Ludolph Lehmus posed his question and popular mathematics magazines are still publishing articles with new proofs of the Steiner-Lehmus Theorem or its generalizations. For example, the October 2001 issue of *The American Mathematical Monthly* carried an article called *Other Versions of the Steiner-Lehmus Theorem*. We can certainly say that for amateur geometers the legacy of Ludolph Lehmus lives on !

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You might be a mathematician if.....
you know by heart the first fifty digits of π;
you have calculated that the World Series actually diverges;
you are sure that differential equations are a very useful tool;
when you say to a car dealer *"I'll take the red car or the blue one"*; you must add *"but not both of them"*.

A Contest Problem Revisited

Professor Y. Nought
Mathematics Dept

In the last issue of MathLinks the solution of a rather simple, but interesting, combinatorial problem was presented as a means of introducing a more challenging problem of a similar nature, offered in the form of a **CONTEST.** We were surprized to find that **NO ENTRIES TO THE CONTEST WERE RECEIVED.** What??!! It was suggested that we try again, so I said to myself "Why not, Y. Nought?"

The challenge issued at that time stands; the rules remain the same as previously stated, **with the deadline for submission of entries extended until 4:00 p.m. Friday 24 May 2002**; the prize for the winning entry also remains unchanged, namely, one copy of each of the two textbooks currently used in the first year calculus (136.150/170) and linear algebra (136.130) courses offered by the Department of Mathematics at the U of M:

Calculus: A Complete Course by Robert A. Adams
Elementary Linear Algebra by StewartVenit and Wayne Bishop.

For a complete set of rules (with the above change in deadline date being noted), please refer to the previous edition of MathLinks.

THE PROBLEM:

Let C be a fixed circle drawn in the plane. Let D be the disk enclosed by this circle. Let n and N denote fixed, but arbitrary, positive integers as defined below:

Let n denote the number of distinct points to be chosen on the boundary curve C of the disk D, and suppose that each of these points is connected to each of the remaining chosen points by a straight line segment. (We shall henceforth assume that $n \geq 2$ *.)*

Let N denote the maximal number or regions into which the disk D is subdivided by the above set of lines joining the chosen n points.

The problem may be expressed simply as

"*Find N as a function of n, denoted as* $N(n)$ ".

For comments concerning the use of the word "*maximal*" in the definition of N , as well as some hints at how one might solve this problem, once again the reader should refer to the previous edition of MathLinks. Additional hints are given below.

HINTS FOR SOLUTION OF THIS PROBLEM:

1. Experimentation is a useful tool in "getting a handle on the problem". To this end, diagrams in the cases when n = 2, 3, 4 and 5 are shown in ***FIGURE 1*** which is found on page 7.

These diagrams allow us to construct the following table

n	$N(n)$	$\Delta N(n)$
2	2	
3	4	2
4	8	4
5	16	8

in which we have adopted the notation that $\Delta N(n)$ denotes the change in N when the n^{th} chosen point is added to the circle C (eg., $\Delta N(3) = 2$, $\Delta N(4) = 4$, $\Delta N(5) = 8$ etc.) ,

so that we may write in general that

$$N(n) = N(n-1) + \Delta N(n) \quad \text{for } n \geq 3 \;.$$

2. Although the above experimentation provides some insight into the problem, the difficulty lies in determining a formula for the change $\Delta N(n)$ in N when the n^{th} chosen point is included on the circle C . To accomplish this, we may, without loss of generality, relabel the chosen points so that their labels increase as we move around C , as depicted in ***FIGURE 2*** shown on page 7.

It should be noted that the line segments joining the various pairs of points have not been included in this diagram. Assuming that all the line segments joining all of the various pairs of points labelled "1" , "2" , ... , "(n-1)" have been drawn, consider the general problem of determining the number of new regions $\Delta N(n)$ added when the last point added (labelled "n") is connected to the k^{th} point added (labelled "k"), with k taking on each of the preceding label values $k = 1, 2, ..., (n - 1)$. When one adds all of these new regions together the value of $\Delta N(n)$ is determined.

A COMMENT: It is very tempting, based on the results in the above table, to make a conjecture that $\Delta N(6) = 16$. THIS IS HOWEVER NOT CORRECT, as a simple "experiment" will confirm; the correct result is $\Delta N(6) = 15$.

3. If the above procedure has been followed, your expression for $\Delta N(n)$ will involve various finite "series", which include "the sums of the various integer powers of the first ℓ positive integers". For your convenience and reference, the sums of various of these series are shown below:

"the sum of the first ℓ positive integers" : $\sum_{k=1}^{\ell} k = 1 + 2 + \ldots + \ell = \frac{\ell(\ell+1)}{2}$

" the sum of the squares of the first ℓ positive integers" : $\sum_{k=1}^{\ell} k^2 = 1^2 + 2^2 + \ldots + \ell^2 = \frac{\ell(\ell+1)(2\ell+1)}{6}$

"the sum of the cubes of the first ℓ positive integers" : $\sum_{k=1}^{\ell} k^3 = 1^3 + 2^3 + \ldots + \ell^3 = \frac{\ell^2(\ell+1)^2}{4}$

"the sum of the fourth powers of the first ℓ positive integers" : $\sum_{k=1}^{\ell} k^4 = 1^4 + 2^4 + \ldots + \ell^4 = \frac{\ell(\ell+1)(2\ell+1)(3\ell^2+3\ell-1)}{30}$.

It is noted that these formulae express the sums of the integer powers of the first ℓ positive integers as simple polynomials in ℓ. You should probably look for a reference book which will confirm the validity of the above formulae. On the other hand, for those of you who are familiar with Mathematical Induction, you could use the Principle of Mathematical Induction (which I sometimes call the Principle of Mathematical Intimidation), in order to prove these results.

4. Having found the correct formula for $\Delta N(n)$ as a polynomial in n, using the above procedure, one may then write,for $n \geq 3$, that

$$\begin{aligned} N(n) &= N(n-1) + \Delta N(n) \\ &= N(n-2) + \Delta N(n-1) + \Delta N(n) \\ &= N(n-3) + \Delta N(n-2) + \Delta N(n-1) + \Delta N(n) \\ &\ldots \\ &= N(2) + \sum_{k=3}^{n} \Delta N(k) \end{aligned}$$

Once again one may use the formulae of hint 3 in order to rewrite this series as a polynomial in n .

5. Clearly the procedure outlined above is not the only means by which we can derive the correct formula for the function $N(n)$. However, this method involves nothing more than some relatively straight-forward counting (or if you prefer, combinatorial) arguments, together with some (tedious, but manageable) algebraic manipulation. If you are careful, you should have no severe difficulties.

Good luck!! We look forward to receiving your submissions. The winner(s) will be announced in the next issue of MathLinks.

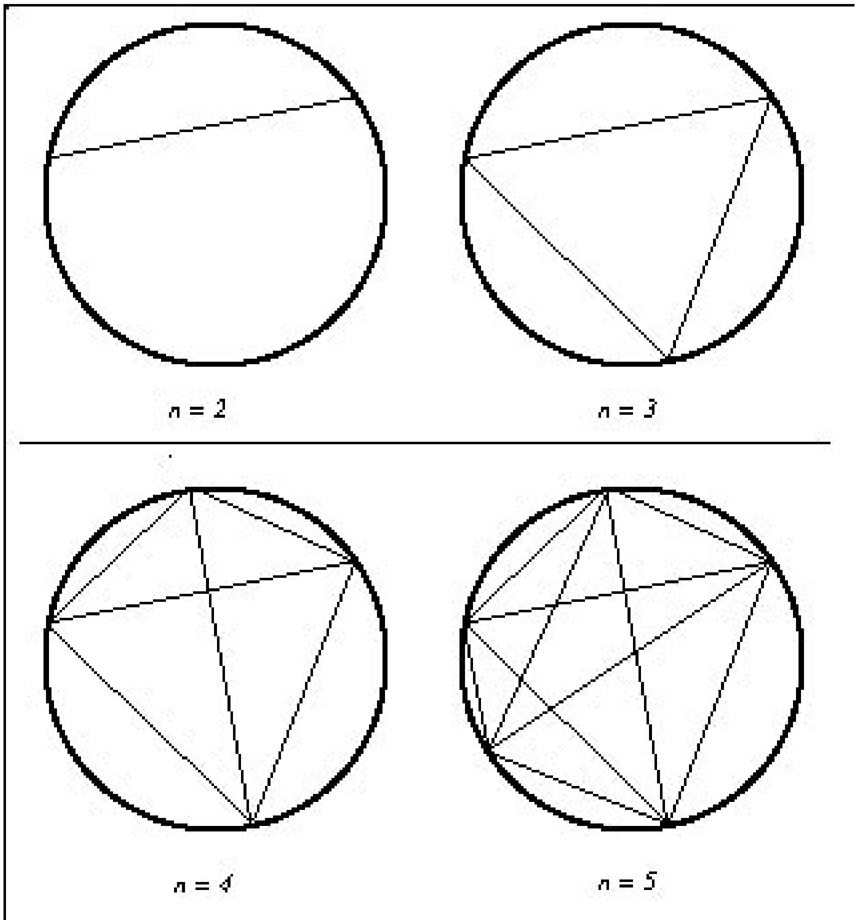

FIGURE 1

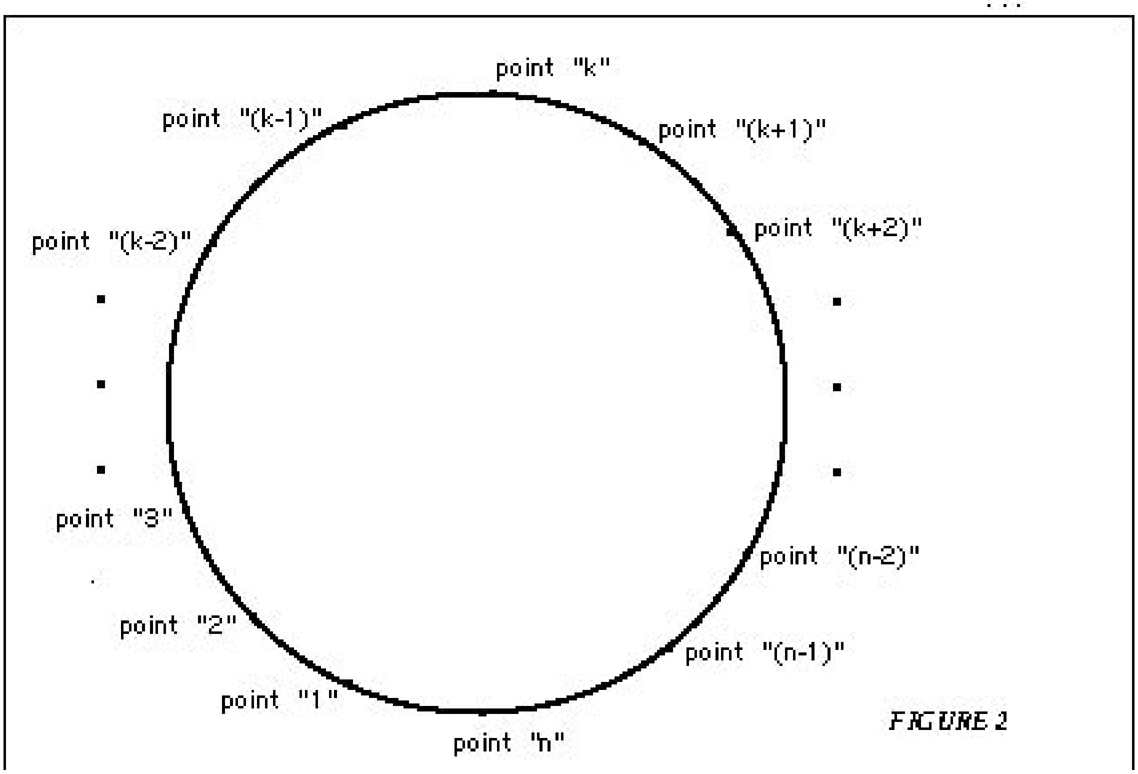

FIGURE 2

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Did You Know????

Florence Nightingale was the first woman to be elected a Fellow of the Royal Statisitical Society.

Florence Nightingale is mainly known for her work as a nurse during the Crimean War in military field hospitals. However, what is not well known was her love of mathematics, in particular statistics, which played a very important part in making her such a famous person.

Born to a wealthy English family in 1820, Florence received the benefit of a higher education which included arithmetic, geometry and algebra. She was influenced by Adolphe Quetelet, a Belgian scientist who applied statistical methods to data involving areas such as moral statistics and social sciences.

Against the wishes of her family, she became interested in social issues, in particular nursing. Although nursing was not considered a suitable activity for a well-educated woman she received training as a nurse in Egypt, Germany and Paris.

The year 1854 saw the start of the Crimean War when Britain, France and Turkey declared war on Russia. When British medical facilities came under heavy criticism in the newspaper, the British Secretary for War asked Florence to become an administrator to oversee the introduction of nurses to military hospitals in Turkey.

Due to extremely unsanitary conditions diseases such as cholera and typhus were rampant in the hospitals. While in Turkey, she collected data, organizing a record keeping system which was used to improve hospital condition ematical ability became evident when she used this information to calculate the hospital mortality rate. This statistical data was used to create her Polar Area Diagram, or as she preferred to call them "coxcombs." These gave a graphical representation of mortality figures during the Crimean War (1854-56).

When peace was declared, Nightingale continued her efforts for sanitary reform in military hospitals using statistics to illustrate the need. Her efforts led to the Royal Commission on the Health of the Army. In 1858, she became the first woman to be elected to be a Fellow of the Royal Statistical Society.

Florence Nightingale transformed nursing into a responsible and respectable career for women. She is also responsible for 200 books, reports and pamphlets all in an effort to improve health standards.

She became an honorary member of the American Statistical Association in 1874. She also became the first woman to receive the Order of Merit from Edward VII in 1907.

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PROBLEM CORNER

D. Trim

Dear Readers:

Welcome back to the PROBLEM CORNER. Here is the problem from the last column and its solution.

The floor function, or sometimes called the greatest integer function, is denoted by $\lfloor x \rfloor$. It is defined to be the largest integer not greater than *x* itself. For example, $\lfloor 4.2 \rfloor = 4$, $\lfloor 3 \rfloor = 3$, and $\lfloor -2.3 \rfloor = -3$. Show that the equation $x^{\lfloor x \rfloor} = 9/2$ cannot have a positive rational solution. Recall that rational numbers are those that can be expressed as quotients of integers.

Certainly *x* must be less than 3 since $x^{\lfloor x \rfloor} \geq 27$ when $x \geq 3$. In addition, *x* must be greater than 2 since $x^{\lfloor x \rfloor} \leq 4$ when $x \leq 2$. Suppose we set $x = 2 + b$, where $0 < b < 1$. Substituting this into the equation gives $9/2 = (2+b)^2$ which reduces to the quadratic equation $2b^2 + 8b - 1 = 0$. Solutions are the irrational numbers $b = \left(-4 \pm 3\sqrt{2}\right)/2$. Since $x = 2 + b$, it follows that *x* itself is not rational.

I received my first submission from a student. ***Mr. Ryan Holm*** from ***St. Ignatius High School*** in ***Thunder Bay*** submitted essentially the above solution to the floor function problem. ***Congratulations Ryan!*** I look forward to more of your solutions and hopefully solutions from other students as well.

Here is your problem for next time:

If we select four different digits from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, they can be arranged to produce 24 different four-digit numbers. Suppose that these 24 numbers are added to produce a sum. What is the maximum number of distinct primes that will divide every such sum?

Let me encourage you to send a solution to:

S. Kangas,
Department of Mathematics,
The University of Manitoba,
Winnipeg, MB R3T 2N2

I will look at all submissions and print the names and schools of persons who solve the problem correctly and present it in a reasonable way .

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